

参赛队员姓名: Andrew Chixiao Yang

中学: Holderness School

省份: New Hampshire

国家/地区: U.S.A / East Coast

指导教师姓名: 高峰

论文题目: Optimization Modeling of Retirement  
Age For Urban Population in China

# CONTENT

<b>ABSTRACT</b>	<b>3</b>
<b>1. INTRODUCTION</b>	<b>3</b>
1.1 Background	3
1.2 Previous Research	4
<b>2. INNOVATIONS</b>	<b>5</b>
2.1 Significance	5
2.2 Models	6
2.3 Analysis	7
<b>3. MODELS</b>	<b>7</b>
3.1 Hypothesis	7
3.2 Model Construction	8
3.3 Parameter Selections	12
<b>4. ANALYSIS</b>	<b>13</b>
4.1 Optimal Retirement Age for Urban Population in China	13
4.2 Optimal Retirement Age in terms of specific cities in China	14
4.3 Sensitivity analysis on different parameters	16
<b>5. CONCLUSION</b>	<b>20</b>
5.1 Results and suggestions for policy makers	20
5.2 Inadequacy and potential improvements	21
<b>BIBLIOGRAPHY</b>	
<b>ACKNOWLEDGEMENT</b>	

# Optimization Modeling of Retirement Age

## For Urban Population in China

Andrew Chixiao Yang  
*Holderness School, Plymouth, U.S.A*

**Abstract:** This paper is dedicated to finding the optimal retirement age for individuals living in urban areas of China in the advent of population aging and the scarcity of pension fund in China. By constructing a model using utility-maximizing rules combined with the three-state model of health relative to specific genders and cities, this paper is able to find the optimal retirement age for male and female in different cities of China and in the nation as well. After applying the Sensitivity analysis for different parameters, this paper is able to suggest cities to enact policies based on individuals' life expectancy age and pensions received.

**Keywords:** Delayed Retirement; Individual Utility Maximization; Three-state Model

## 1. Introduction

### 1.1 Background

Retirement, first introduced in Germany in 1889, is a relatively new idea to human societies<sup>[1]</sup>. In most countries, an individual is expected or required by the government to cease working at a certain age, which is known as the retirement age. Differed from country to country, retirement age is determined by policymakers based on demography, health, life expectancy, aging costs, etc. of their particular countries. In China, the prototype of the current retirement age policy was first enacted by the State Council of China in the “Labour Insurance Regulations of the People’s Republic of China” in 1951, proposing the required age for males to be 60 and 50 for females<sup>[2]</sup>. The current retirement policy in 2019 was settled in 1978 in “State Council Provisional Regulations on Retirement and Resignation of Workers,” which set for a retirement age of 50 for female workers, 55 for female cadres, and 60 for males<sup>[3]</sup>.

However, the retirement policy enacted in the 70s is encountering multiple issues which the Chinese society has to face with nowadays. One of the crucial issues is Population Aging.

According to data from WHO, the estimated life expectancy of an average Chinese has grown from 66.84 years in 1980 to almost 76 years in 2019, which means Chinese people are living longer than ever and this number is probably going to continue to grow<sup>[4]</sup>. Entering the 21st century, the elderly population in China has increased exponentially. According to the “Statistical Communique of The People’s Republic of China On National Economic and Social Developments” public by the National Bureau of Statistics of China in 2015, China has over 220 millions of elderlies who are over 60 years old, ranking first in the world<sup>[5]</sup>. Consisting of 16.1% of the whole population, the elderly population in China is estimated to swell at an alarming rate, as anticipated by Zhai et al. to grow over 34% of the Chinese population in the middle of the 21st century<sup>[6]</sup>. Population aging has become a serious issue concerning the Chinese retirement policy. As the population grows older in general, the proportion of the populations in the labor force will be reduced. As a result, less people are paying pensions, while more are receiving pensions after their retirement. Zhang et al. have anticipated a gap of over 1.8 thousand billion yuan in China’s pension fund by 2025<sup>[7]</sup>. Another societal issue concerning retirement age in China is the inequality in the pension fund. The so-called “Double-Track System” in pension fund has caused a considerable gap between the employees from public and government institutions and private-owned enterprise<sup>[8]</sup>. It was generally believed that the gap was about 300% to 500%, causing those who worked at private-owned companies or those who receive pension fund as individuals get far less money than what they used to earn at work. With increasing inflation rate in the economy, the pensions received by those who worked at private-owned companies were unable to support their daily expenses in China after retirement, especially in metropolitan areas like Beijing, Shanghai, and other first-tier cities. Considering the ineffectiveness of the retirement policy in contemporary settings, the Third Plenary Session of the 18th CPC Central Committee in 2013 proposed an idea of potential laws in the future for delayed retirement age in China<sup>[9]</sup>.

## 1.2 Previous Research

As for finding the optimal retirement age, there is a notable difference between the methods Western and Chinese scholars approach the issue. Because of the policy difference between China and other countries—while most western countries have a rather lenient retirement policy, China has enacted a strict retirement policy which stresses uniformity—western scholars tend to approach optimal retirement age concerning more about individuals differences in their choice of retirement. For example, B.Tacchino researched and listed over 25 factors which could impact an individual's optimal retirement age<sup>[10]</sup>. On the other hand, Chinese scholars tend to approach the issue qualitatively and rarely did any researchers quantitatively approached the issue with vast data<sup>[11]</sup>. Lei et al. had constructed a model based on DB to maximize pension fund received<sup>[12]</sup>. However, the paper failed to consider the differences each individual received in their respective pension funds. Zhang et al. constructed a model based on the principle of maximizing social welfare without considering individual utility and benefits<sup>[7]</sup>. Therefore, on the topic of finding the optimal retirement age, especially in China, innovations are left to be made in finding the optimal retirement age.

## **2. Innovations**

### **2.1 Significance**

On the edge of the process of population aging in China and the inequality and scarce nature of pension fund in the country, this paper expects to provide a more practical approach which could address most potential variables in determining the optimal retirement age in China. Instead of focusing on social welfare on a macro level, this paper is going to focus on individuals in China and try to find the optimal retirement age, which suits their best interests. However, unlike previous research, which focused on the financial aspect, such as maximizing the pension fund received, this paper focused mainly on the utility obtained by each individual on optimal retirement age. Utility means satisfaction, and for most individuals around the age of retirement, satisfaction means better health. Therefore, this paper builds up the approach mainly on one's health status and healthcare expenses. It is believed that trying to maximize one's utility based on

their health status means minimizing the risks those elderly have to face when experiencing with retirement, and for those at an age around retirement, reducing the risks of physical unhealthiness is their top priority.

## 2.2 Models

The model this paper employed was based on the three-state optimal retirement model for utility-maximizing modified by Gao et al.<sup>[13]</sup> The innovation made by this paper will be discussed first while the specific model will be presented in the next section. This paper had innovated the former model in the following five aspects. First, instead of setting the chances of one's health status that could negatively impact their life at a certain age as a constant, this paper employs the data from China's 1% National Sample Census in 2015 and plots an exponential function using Excel for both male and female population concerning their health status and age<sup>[14]</sup>. Second, concluding from Lei et al. that retirement will impact one's health status by a constant parameter  $\beta$ , which differs from male and female, this paper modifies the model presented by Gao et al. on health and age<sup>[15]</sup>. Third, this paper divides the utility-maximizing model into three parts, adding randomness to the age when one's having a health issue, which is more suitable for real-world applications, because no one will ever know when his or her body is going to get wrong and will force him or her to retire. Fourth, this paper uses a quantitative approach by applying different data and parameters, such as average income and average health care spending from the National Bureau of Statistics in China, and the life expectancy in different cities from local Government and Health Commission. In order to consider the inequality in pension funds, the paper adopts the average pension fund received in specific cities from the National Bureau of Statistics. Last but not the least, this paper uses the programming software R to find the maximum utility and optimal age, instead of using derivatives as previous papers did, so that this paper is able to conduct a quantitative approach as mentioned above with more parameters in the model.

## 2.3 Analysis

This paper has made four innovative approaches in the analysis section. First, when analyzing the data and results obtained, this paper limits the data to the urban population, since they are mostly the targets of the retirement policy. Also, the paper considered the optimal retirement age for both female and male separately, because female and male have a different life expectancy, health status at a different age, and in the most part, unequal pay, which is a global issue nowadays; therefore it is essential to analyze the two groups separately. Besides genders, this paper also recognizes the variables of income, life expectancy, healthcare expenses, and pension fund received on one's retirement age, by conducting multiple sensitivity analysis modifying each parameter while keeping others unchanged, this paper can find which parameter has an obvious relationship with the optimal retirement age. Lastly, this paper has also applied data from specific cities which are different from each other, in order to find whether the national retirement policy could also apply to and be suitable for specific cities as well.

### **3. Models**

#### **3.1 Hypothesis**

The model is constructed based on the following hypothesized scenarios:

- a. There are three states in one's life: 1. The Working State in which one's personal income is solely based on the salary received, and their personal utility is gained through consumption determined by their personal income. 2. The Retirement State in which one's personal income is solely based on the pension fund received, and their personal utility is also gained through consumption determined by their income. 3. Early Retirement State, a state of life where one is forced to retire from work because of his or her unpromising health condition. In the Early Retirement State, one's solely income is the pension fund, and there's a constant consumption on healthcare expenses, which might bring negative utility.

- b. Under the state of working, one's annual income is changing with the rate wage changes, which is a combination of the rate of return on wages, nominal interest rate, and inflation rate.
- c. When someone's health status begins to show negative impact on one's life, whether or not one could still take care of oneself, he or she will enter Early Retirement State. The healthcare expenses changes as the same rate wage changes.
- d. The pension fund received is the same as that of every year after retirement.
- e. The utilities from healthcare expenses and personal income are all constant.
- f. The decision of entering into retirement would negatively impact one's health status.

### 3.2 Model Construction

The utility functions for an individual under Working State, Retirement State and Early Retirement State are  $u_1, u_2, u_3$ , satisfying the *Inada* condition:

$$u_1' > 0, u_1'' < 0, u_2' > 0, u_2'' < 0, u_3' > 0, u_3'' < 0.$$

The parameters of the utility functions are the following:

$C_1$  : The consumption at Working State;  $C_2$  : The consumption at Retirement State;  $C_3$  : The consumption when the health status showed negative impact on one's life. Then the utility functions **(1)** are:

$$u_1(C_1) = \ln(C_1)$$

$$u_2(C_2) = \ln(C_2)$$

$$u_3(h) = -\ln(C_3)$$

Suppose  $\theta$  is one's constant consumption utility;  $\alpha$  is one's constant utility on healthcare expenses;  $W$  is one's total income from salary;  $V$  is one's total pension fund income;  $h$  is one's total expenses on health care, then the consumption functions **(2)** are:

$$c_1 = \theta W$$

$$c_2 = \theta V$$

$$c_3 = \alpha h$$

The Utility function (3) therefore are:

$$u_1 = \ln(\theta W)$$

$$u_2 = \ln(\theta V)$$

$$u_3 = -\ln(\alpha h)$$

Suppose  $T$  is age of death;  $R$  is the optimal retirement age;  $T_0$  is the age when one's health status showed negative impact on one's life;  $\beta$  is the risks of one becoming sick switching to retirement state. Suppose  $h_0$  is the annual healthcare expenses;  $t_1$  is the age one started working;  $r$  is the real interest rate;  $\pi$  is the inflation rate;  $\varepsilon_0$  is the rate of return on wages,  $\varepsilon$  is the rate of change on wage.

$$\varepsilon = \varepsilon_0 + \pi + r.$$

The chances of one experienced the negative health status before retirement ( $T_0 < R$ ) is  $P_1$ ; The chances of one experienced the negative health impact after retirement but before death ( $R < T_0 < T$ ) is  $P_2$ ; The chances of one experienced a negative impact after death, in another word, remained health throughout life ( $T_0 > T$ ) is  $P_3$ .

$$P_1 + P_2 + P_3 = 1$$

Applying data of the health status and age from *China's 1% National Sample Census in 2015*, this paper is able to find the exponential fit: The chances that one is negatively impacted by his or her health status at age  $x$  is  $ke^{\lambda x}$ , where  $k$  and  $\lambda$  are constants. The chances that one is negatively impacted by his or her health status at age  $(x, x+dx)$  is  $k\lambda e^{\lambda x} dx$ . Hence,

$$P_1 = ke^{\lambda x} - \beta \quad (4)$$

$$P_2 = ke^{\lambda R} + \beta \quad (5)$$

$$P_3 = 1 - P_1 - P_2 \quad (6)$$

The schematic diagram showing  $P_1$ ,  $P_2$ ,  $P_3$  are shown below:

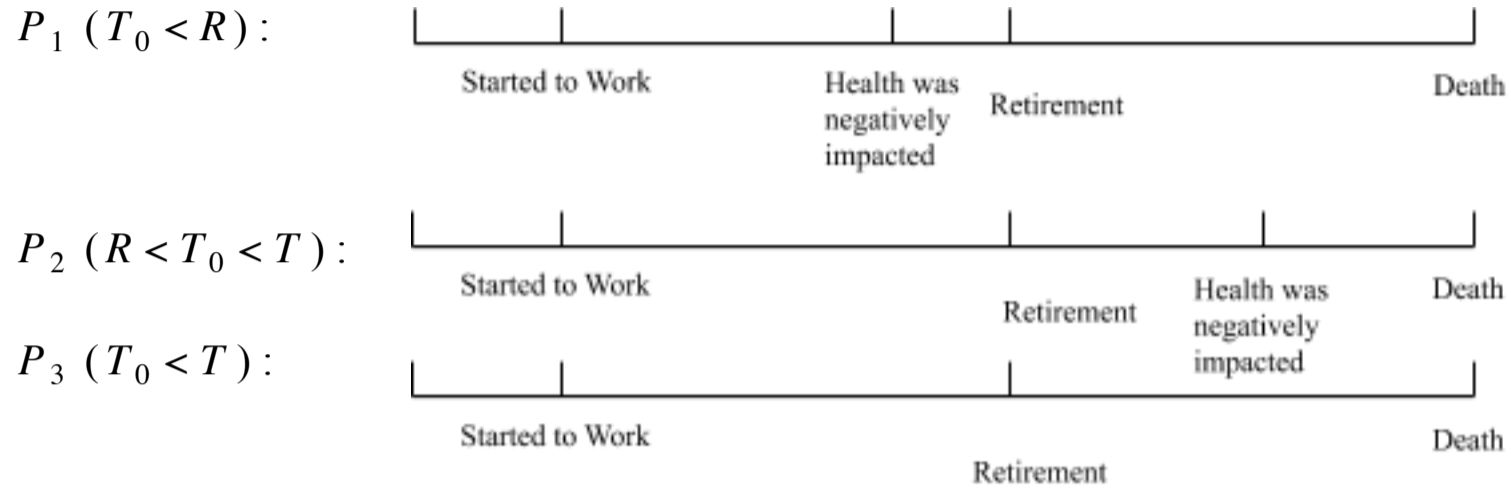


Image 1: The Three State of Negative Impact on One's Health<sup>[13]</sup>

A. When one retires early because of his or her health status ( $T_0 < R$ ), the total healthcare expenses of his or her life is  $h_1$  at a retirement age  $R$  (7):

$$h_1 = h_0(1 + \varepsilon)^{R-T_0} + h_0(1 + \varepsilon)^{R-T_0-1} + \dots + h_0 + \frac{h_0}{1+\varepsilon} + \dots + \frac{h_0}{(1+\varepsilon)^{T-R}}$$

$$= h_0(1 + \varepsilon)^{R-T} \frac{(1+\varepsilon)^{T-T_0+1} - 1}{\varepsilon}$$

Retiring at  $R$  age, one has a total income of  $W_1$  (8):

$$W_1 = W_0(1 + \varepsilon)^{R-t_1} + W_0(1 + \varepsilon)^{R-t_1-1} + \dots + W_0(1 + \varepsilon)^{R-T_0}$$

$$= W_0(1 + \varepsilon)^{R-T_0} \frac{(1+\varepsilon)^{T_0-t_1+1} - 1}{\varepsilon}$$

The total pension fund received  $V_1$  (9):

$$V_1 = V_0(1 + \varepsilon)^{R-T_0} + V_0(1 + \varepsilon)^{R-T_0-1} + \dots + V_0 + \frac{V_0}{1+\varepsilon} + \dots + \frac{V_0}{(1+\varepsilon)^{T-R}}$$

$$= V_0(1 + \varepsilon)^{R-T} \frac{(1+\varepsilon)^{T-T_0+1} - 1}{\varepsilon}$$

B. When one shows negative health status after normal retirement ( $R < T_0 < T$ ), the total healthcare expenses of his or her life is  $h_2$  **(10)** at a retirement age  $R$  :

$$h_2 = \frac{h_0}{(1+\varepsilon)^{T_0-R}} + \frac{h_0}{(1+\varepsilon)^{T_0-R+1}} + \dots + \frac{h_0}{(1+\varepsilon)^{T-R}}$$

$$= h_0(1 + \varepsilon)^{R-T} \frac{(1+\varepsilon)^{T-T_0+1}-1}{\varepsilon}$$

The total income  $W$  **(11)** one received since working at  $t_1$  :

$$W = W_0(1 + \varepsilon)^{R-t_1} + W_0(1 + \varepsilon)^{R-t_1-1} + \dots + W_0$$

$$= W_0 \frac{(1+\varepsilon)^{R-t_1+1}-1}{\varepsilon}$$

The total pension fund received  $V$  **(12)** from retiring at age  $R$  to death at age  $T$  is:

$$V = \frac{V_0}{1+\varepsilon} + \frac{V_0}{(1+\varepsilon)^2} + \dots + \frac{V_0}{(1+\varepsilon)^{T-R}}$$

$$= V_0 \frac{1-(1+\varepsilon)^{R-T}}{\varepsilon}$$

C. When one remains healthy throughout life and retired accordingly ( $T_0 > T$ ), no healthcare expenses and its utility will be generated. Only income from work and income from the pension will remain, which is the same as when  $R < T_0 < T$ .

According to the analysis on the three-state model above, the utility function  $E(u)$  at one's optimal retirement age  $R$  **(13)** is:

$$E(u) = P_1(u_1' + u_2' + u_3') + P_2(u_1 + u_2 + u_3) + P_3(u_1 + u_2)$$

Given the random nature of  $T_0$ , plugging in models **(4)-(12)** into **(13)** and getting **(14)**,

$$E(u) = \sum_{t_1 < x < R} ke^{\lambda x} [\ln(\theta W_1) + \ln(\theta V_1) - \ln(\alpha h_1)] + \sum_{R < x < T} k\lambda e^{\lambda x} [\ln(\theta W) + \ln(\theta V) - \ln(\alpha h_2)] + \{(0.03 + ke^{\lambda R})$$

$$[\ln(\theta W) + \ln(\theta V) - \ln(\alpha h_2)] + [\ln(\theta W) + \ln(\theta V)](1 - ke^{\lambda T} - 0.03)\}$$

The analysis section is based on the model above using programming software  $R$ .

### 3.3 Parameters Selections

Model **(14)** was built on the premise that some of the parameters are city-specific, while others could be applied throughout the country. The idea of  $\varepsilon_0$  the rate of return on wages was inspired from Li, who proposed the integration of the change of wages into model constructions<sup>[16]</sup>. This paper set  $\varepsilon_0 = 1\%$  as proposed by Heckman et al.<sup>[17]</sup>. Inflation rate  $\pi = 2\%$  in CPI, and real interest rate  $r$  is usually  $1\%$ .<sup>[13]</sup> As mentioned in previous sections, the construction of  $P_1, P_2, P_3$  is based on the data obtained from *China's 1% National Sample Census in 2015*. After using *Excel*,  $k$  is found to be 0.0005 and 0.0004 for male and female respectively;  $\lambda$  is found as 0.0737 and 0.0776 for male and female as well. As the curve of age and health status is plotted below:

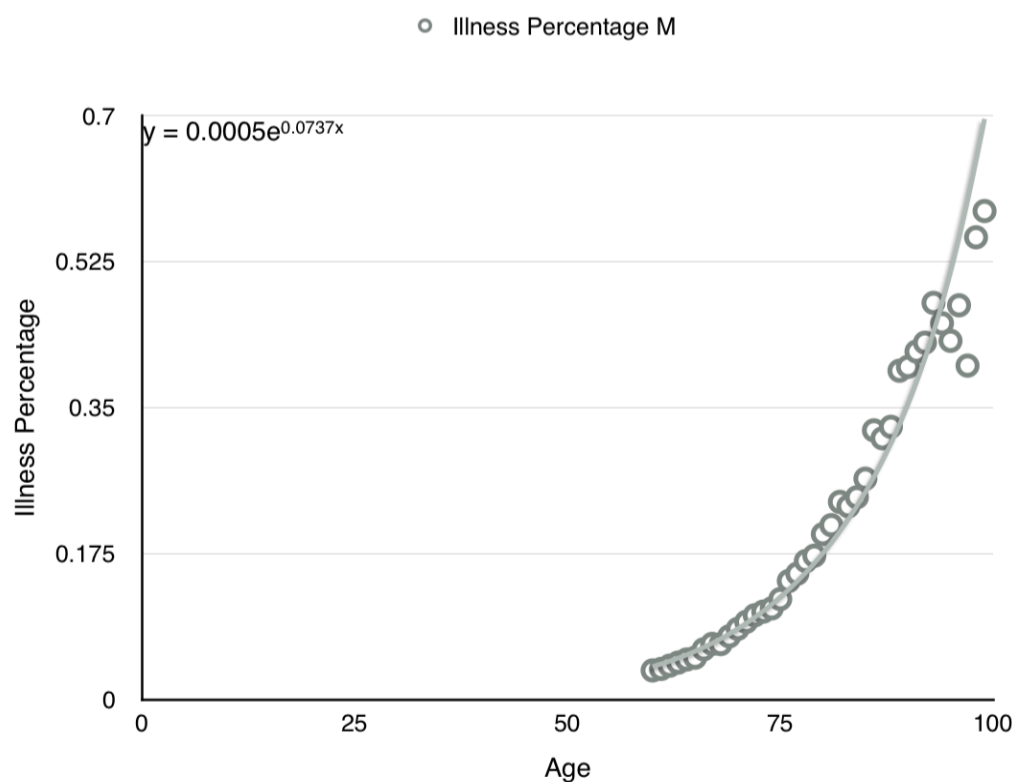


Image 2: male

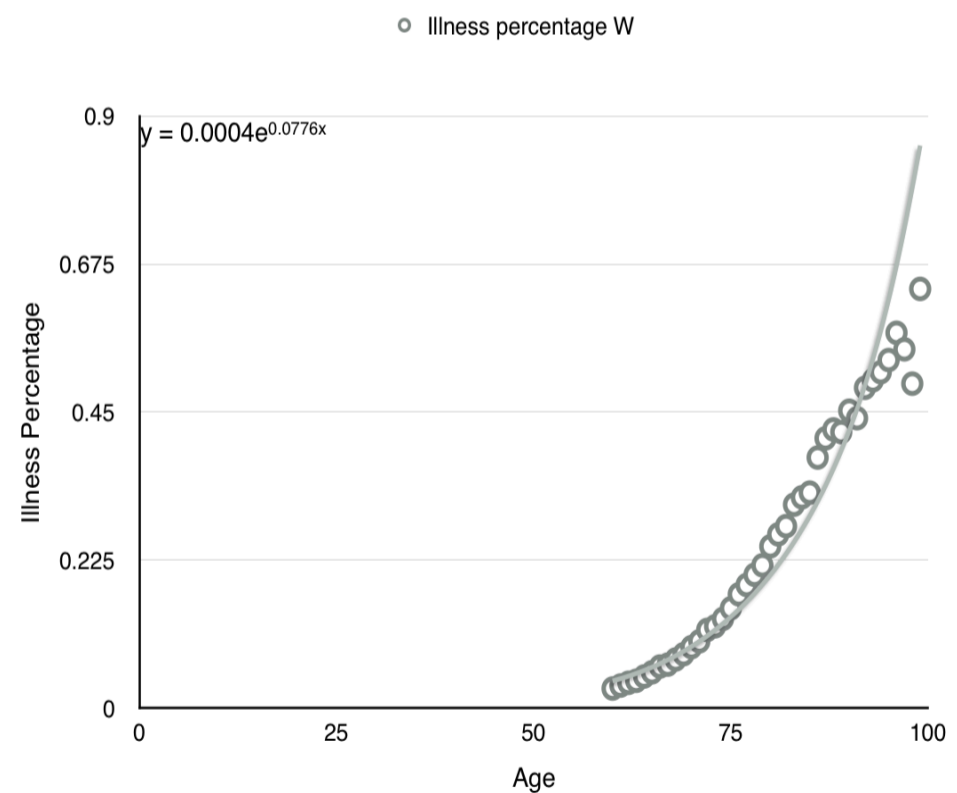


Image 3: Female

$\beta$ , the chances one's health could be negatively impacted from retirement is drawn from Lei et al. 's research on the impact of retirement on one's health status as -0.003 for male and -0.001 for female.<sup>[12]</sup>  $t_1$ , the age one begins to work for most situations, is 20 years old.<sup>[13]</sup>

The rest is obtained in a specific setting, typically in the context of specific cities or genders. While the specific number of the following parameters will be discussed in the next section in analysing results, this section will explain the origins of the following parameters. Income  $W_0$ , pension  $V_0$ , Healthcare expenses  $h_0$ , and as well as  $\theta$ , one's constant consumption utility, and  $\alpha$ , one's constant utility on healthcare expenses, were found in *Chinese Statistical Yearbook in 2018* for data concerning urban population on a national level, while data for specific cities could be found from their respective local government website.<sup>[18]</sup>  $T$ , the life expectancy obtained its data from the WHO and local Health Commissions.

## 4. Analysis

### 4.1 Optimal Retirement Age for Urban Population in China

#### a. Males

According to data from the Bureau of Statistics of China, Male population living in urban areas of China had an average life expectancy of 75 years old in year 2018. The average income of an individual earned in urban areas according to the same source was 27906.37931 yuan.

Regardless of the pension fund “Two-Track System” in China, the average pension fund an individual received, calculated from *Chinese Statistical Yearbook in 2018*, was around 7 % of their original wages, not counting other insurance and funds they purchased related to their retirement plan. Healthcare expenses was about 8 percent of one's total consumption. After applying all the parameters with specific data values, this paper found 58.51514 or 59 years old taking the integer as the optimal retirement age for men in urban areas of China. 59 years old is

around 60 years old, which is the current required age for male workers in China. The result had shown there's no change needed for male to delay their retirement on a national level in China.

#### b. Females

Females in China, according to the same statistics has a life expectancy of 77.9 years old.

According to Research on Gender Differences in China's Job Market in 2019, there's an existing Gender Pay Gap in China between female and male workers<sup>[18]</sup>. Female typically earned 78.3% of the wages male earns in the same workplace. After considering this feature for females and applying all the parameters for female into the model with  $R$ , this paper has found 62.5588 or 63(integer form) years old as the optimal retirement age for female in urban China. 63 years old showed a distinctive difference with 50 years old, the former retirement age for females in China. As was shown from the life expectancy data, females typically have a longer life expectancy, and according to *China's 1% National Sample Census in 2015*, females' health status have a relatively slower rate of "decaying" compared to that of males before their 65 years old.

### 4.2 Optimal Retirement Age for specific cities in China

China's economy has boomed at an accelerating rate after entering the 21st century. People's lives have changed rapidly, especially in metropolitan areas. Their income, and life expectancy, the two pillars of their constructed model, are way above the average in the country. Therefore, it is necessary to consider the specific situations of different cities in China in terms of retirement policy.

#### a. Beijing, Shanghai, Guangzhou

Beijing, Shanghai, and Guangzhou are the three most developed cities in China in terms of their economy and healthcare system. According to the Shanghai Health Commission's data in 2019, the average health expectancy in Shanghai has reached 83.63 years old for male and 86 for females, which is far longer than the national average. Also, being the largest city in China, Shanghai's average income has risen to about 65 thousands yuan. Similar situations have

happened in other cities as well, with longer life expectancy and more average income. It is logical to suppose that the optimal retirement age would be different in cities like Shanghai, so this paper applies the data of each city into the model, and the results are shown below:

Cities	Optimal R for Males	Optimal R for Females	Average Income	Life Expectancy Males	Life Expectancy Females
<b>Beijing</b>	69.09838	74.11156	62,361	82.2	84.63
<b>Shanghai</b>	69.00536	77.50163	64,183	83.63	86
<b>Guangzhou</b>	66.6946	75.00896	55,400	81.34	84.93

Table 1: Beijing, Shanghai, and Guangzhou

*(Income: National Bureau of Statistics; Life Expectancy: Local Health Commissions; Healthcare: National Bureau of Statistics; Pension Fund: Pension fund calculator \*Note: Women's income are 78.3% of men's when calculating)*

The optimal age found for the three largest cities in China is far beyond the national average the same model has found in the previous section and the government policy as well. The high life expectancy in each city and the high income received have contributed to the result of delayed retirement age. However, one of the reasons that the delayed retirement age is so “absurdly” older than the national level and what the previous relevant national policies has stipulated is because of the unequal nature of the pension fund. As mentioned in the introduction section, the pension fund received by employees in private-owned companies is so little that it could barely support their lives in substantial metropolitan areas like Beijing, Shanghai, and Guangzhou. As one retires with far less income than when he or she is at work, his or her utility would be reduced substantially since they are unable to sustain the same consumption and utility as they were before retirement in cities like Beijing. However, recent news has shown signs of policies being made to narrow the inequality gap, so the results have justified the idea of delayed retirement in China, especially in Metropolitans, where the government might need specific policies for their residents.

#### b. 1st-tier Cities & Less developed cities

In order to find the cities that could apply the national retirement age found in previous sections, this paper applies data to “relatively less developed” cities as well. Nanjing, reported to have the fastest GDP growth in 2019, is a typical 1st-tier city in China. According to the data from

Nanjing government and Nanjing Health Commission, the city is believed to obtain the similar level of average income and life expectancy with Guangzhou (which is shown above). Nanjing is having an average income of 55,440 and life expectancy of 81.16 and 86.53 for male and female. With no surprise, the optimal retirement age produced by the model was 66.3322 years and 77.90 years for male and female. It is implied that not only the top metropolitans in China but also the 1st-tier cities with similar personal income and life expectancy are better off to change to late retirement according to the model. Chengdu, the capital of Sichuan province, is quite unique as the average income is about 55,200, but the average life expectancy is only 74.92 and 79.64 years old for male and female respectively. After plugging in Chengdu's data into the model, the optimal retirement age was 58.53 and 65.1636 years old, which is close to the national data. This paper also picked two other cities whose average income and life expectancy are close to the national average, i.e., Jiuquan, and Yinchuan, in the western part of China. The results are shown below:

Cities	Optimal R for Males	Optimal R for Females	Average Income	Life Expectancy Males	Life Expectancy Females
<b>Chengdu</b>	58.53411	65.16366	55,200	74.92	79.64
<b>Jiuquan</b>	58.3479	59.24285	35,239	74.8	74.8
<b>Yinchuan</b>	58.34788	59.24281	35,586	74.86	74.86
<b>China Urban areas</b>	58.51514	62.5588	27,906.37931	75	77.9

Table 2: Chengdu, Jiuquan, Yinchuan, and national average

*(Income: National Bureau of Statistics; Life Expectancy: Local Health Commissions; Healthcare: National Bureau of Statistics; Pension Fund: Pension fund calculator \*Note: Women's income are 78.3% of men's when calculating)*

Cities with distinctive average income but similar life expectancy ages are sharing the similar optimal retirement age. This observation has raised the need for further sensitivity analysis in a bid to find the parameters which might have decisive impact the optimal retirement age in the model constructed.

#### 4.3 Sensitivity analysis on different parameters

The sensitivity analysis is based on the existing data from the city of Beijing. By changing the following selected parameters and comparing the results, the following chart tries to identify key factors that might have the most significant impact on calculating the optimal retirement age.

a. Income

Income	Retirement Age M	Retirement Age W	
5000	69.0983	74.1112	
10000	69.0903	74.1121	
20000	69.0938	74.11178	
62361	69.0938	74.1156	Controlled
100000	69.0983	74.11152	
500000	69.0983	74.57408	
1000000	69.0983	74.1114	

Table 3: Income Sensitivity analysis

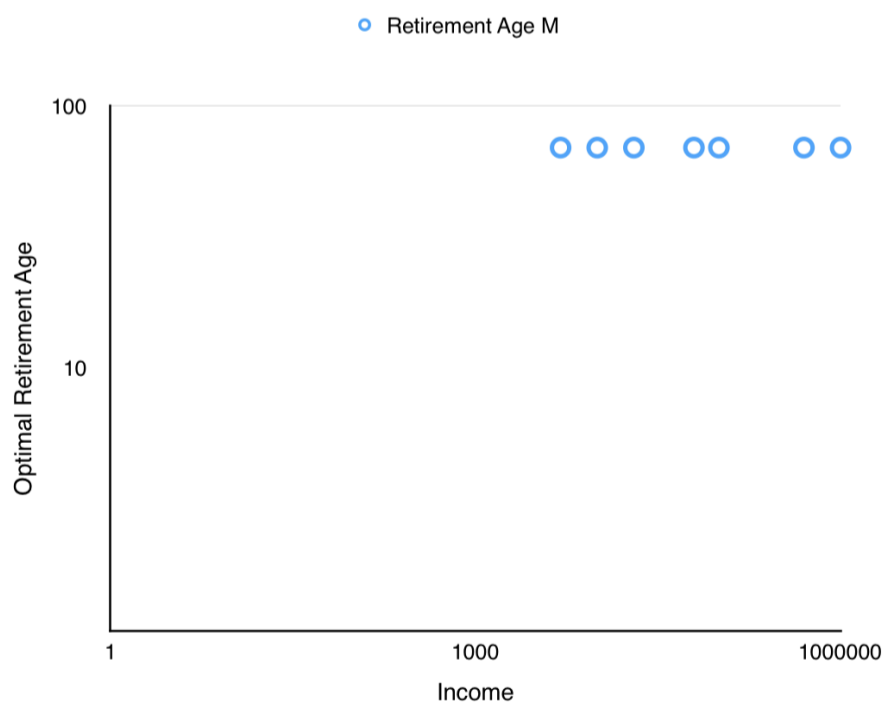


Image 4: Income Sensitivity analysis for male

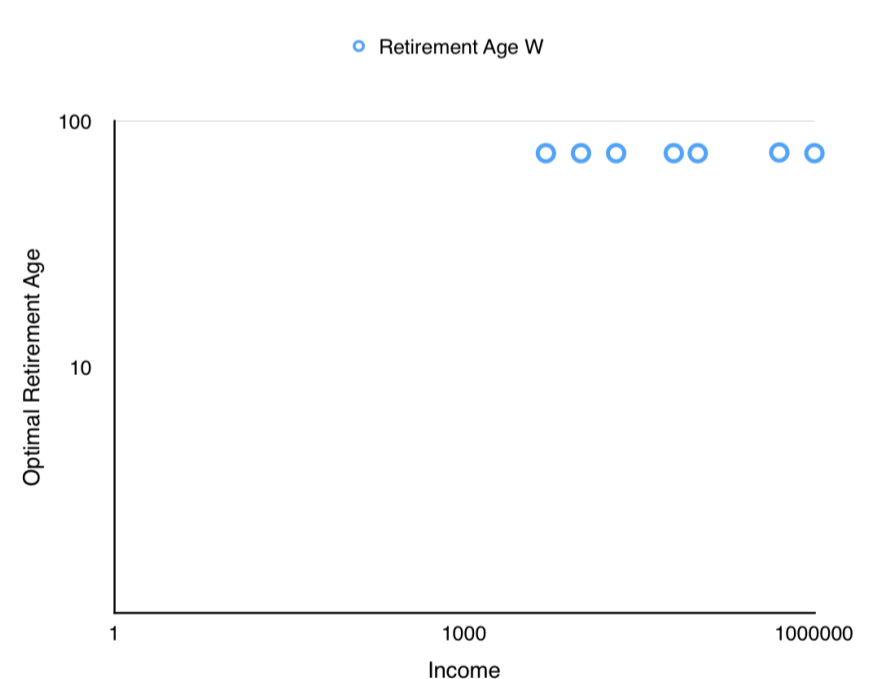


Image 5: Income Sensitivity analysis for female

(Note: Women's income was calculated as 78.3% as men's)

As observed above, there is no significant change in the optimal retirement age as one's income has changed exponentially. The main reason for the results is because the model took the average

pension received after retirement, which according to the pension fee calculator and data derived from *Chinese Statistical Yearbook in 2018*, was about ten percent of the original income received. As a result, as employees have retired from work, they will receive far fewer pension fees and will primarily reduce their utility after retirement, which would make the option of keep working feasible.

b. Pension

Pension Percentage in Income	Optimal Retirement M	Optimal Retirement W	
7%	69.0938	74.1156	Controlled
50%	69.0987	74.1115	
100%	69.0983	74.1115	
120%	68.4653	74.0094	
200%	65.652	71.4026	
500%	62.3568	68.2057	
1000%	59.54923	65.78	

Table 4: Pension Sensitivity analysis

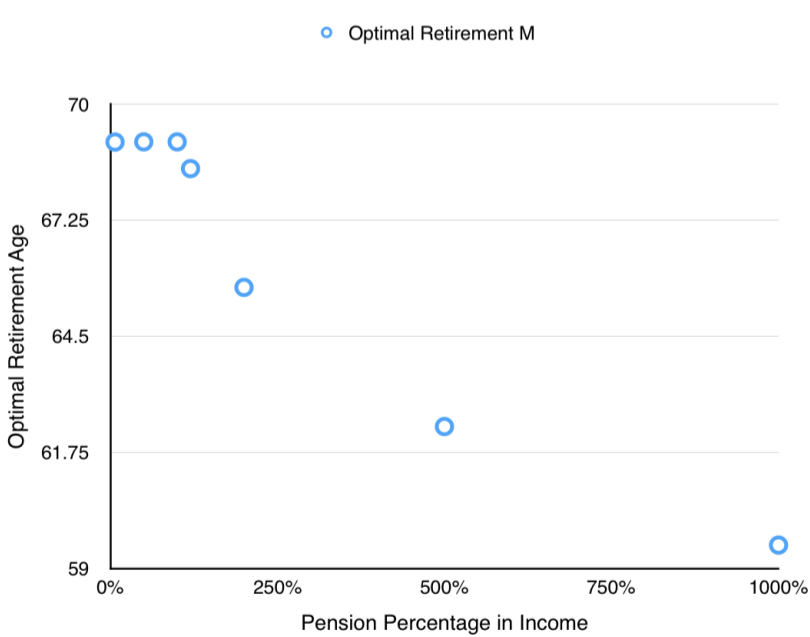


Image 6: Pension Sensitivity analysis for male

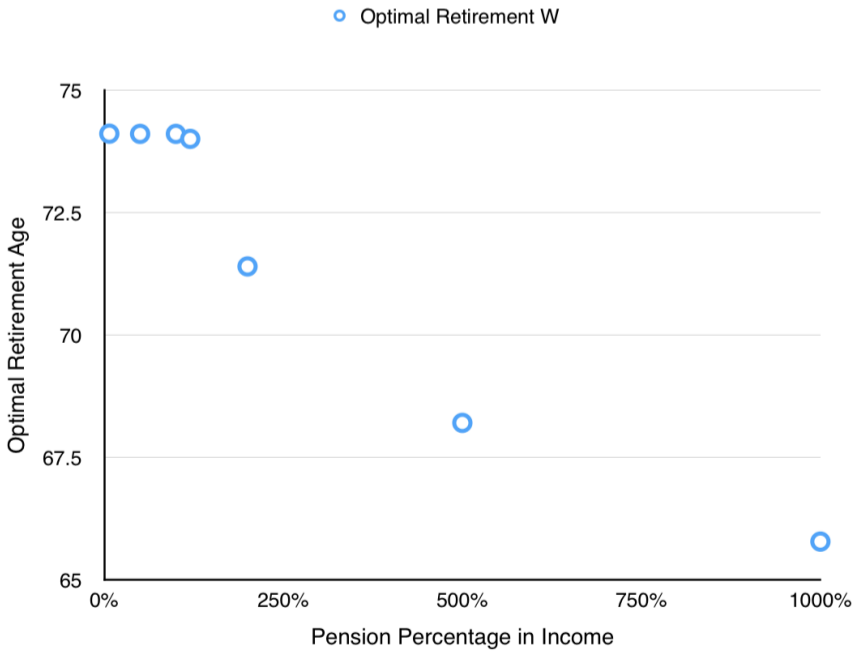


Image 7: Pension Sensitivity analysis for female

(Note: Women’s income was calculated as 78.3% as men’s)

The graphs above showed a tendency that, as pension's percentage relative to income increases, the optimal retirement age suggests to be younger. It is true with what happened in the Sensitivity analysis for income. The reason why the showed tendency happens is because the more pension fees one received after retirement, the more anticipated utility one will possess after retirement as they are receiving far more money than what they used to earn at work. This scenario usually happens for cadres working in state institutions, and has resulted in the legitimacy of the latest policy regarding female cadres who would be required to delay their retirement to 60 years old. The result also suggests the delay could continue to extend to around 65 years old, which could save a large proportion of the pension fund of the country.

### c. Life Expectancy

Life expectancy	Optimal Retirement Age M	Optimal Retirement Age W
73	55.35	56.74
76	59.1	59.4
79	63.23	64.43
82	69.09	69.48
84	73.07	73.03
90	74.13	85.56

Table 5: Life Expectancy Sensitivity analysis

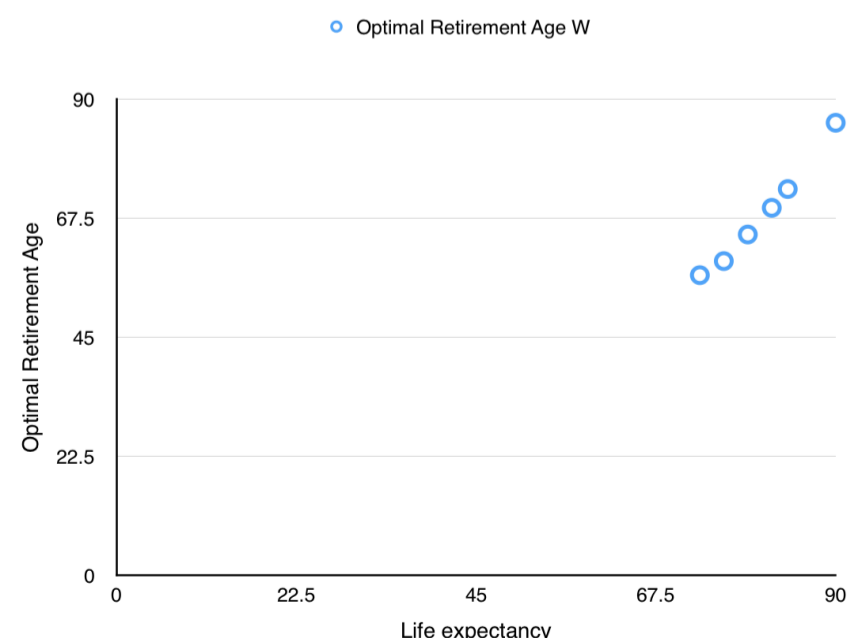
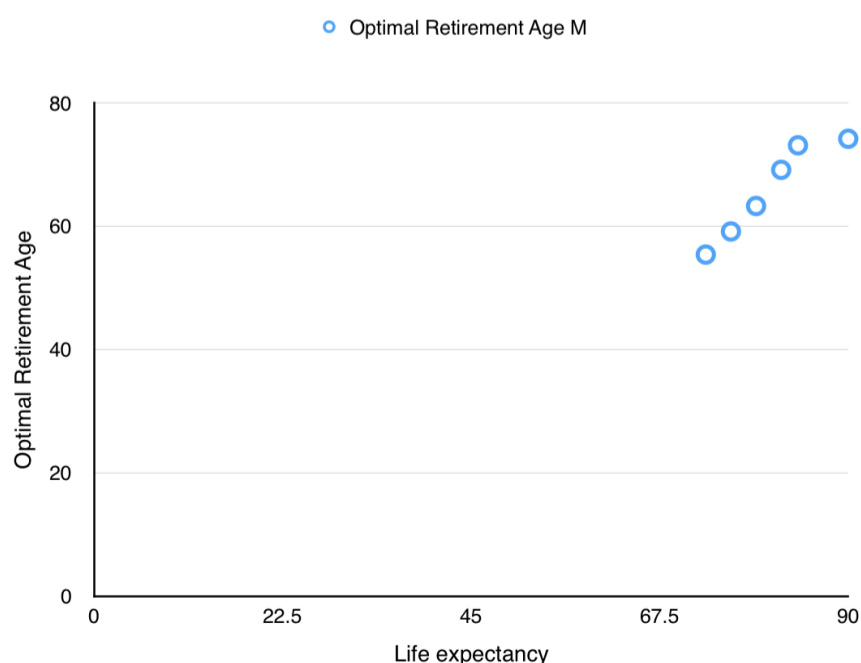


Image 8: Life Expectancy Sensitivity analysis for male

Image 9: Life Expectancy Sensitivity analysis for female

(Note: Women's income was calculated as 78.3% as men's)

The Sensitivity analysis for life expectancy age showed a significant relationship between life expectancy and optimal retirement age calculated. As one's life expectancy gets longer, the optimal retirement age increases as well. Since one is expected to live longer, their anticipated utility for working outweighs the utility for retirement. Retirement usually leads to lower income and a higher chance of having health issues, thus keeping on working may provide higher consumption utility from the higher income received, and longer life expectancy could offer more healthy working years for a higher anticipated utility. According to the table above, women tend to have an older optimal retirement age. This trend explains that women receive fewer wages in general, so they need more working time to obtain the maximized utility before the optimal retirement age.

## 5. Conclusion

### 5.1 Results and suggestions for policy makers

The results obtained from the model have showed a pattern of the delayed retirement for males and females in general, especially for females as they have a longer life expectancy and relatively lower wages. After analyzing data from specific cities in China, this paper found that the national retirement policy might be ineffective in metropolitan areas like Shanghai and other first-tier cities in China. As the original mission of this paper is to find the optimal retirement age, which satisfies individual interests, and each individual lives different lives in different cities, it is hard to find un uniformed retirement policy and optimal retirement age. Considering that the original aim of this paper is to find the retirement age that could best satisfy individual utility, this paper suggests the government enact policy specified to each individual, based on their pension received and life expectancy age. Also, this paper found that for metropolitan areas like Beijing and Shanghai, the inequality incurred by the “Double-track system” should be appropriately addressed. Since the gap between wage and pension fund is so significant for

employees at non-state run institutions and companies, normal retirement may not be the best way to maximize their anticipated utility as compared with the option of keeping on working.

## 5.2 Inadequacies and potential improvements

The Sensitivity analysis has showed no significant correlation between optimal retirement age and income, seemingly implying that the model constructed may still have rooms for modifications or improvements. For example, one's bank deposits may have some effects. If they have made enough money from work, they would not need to worry about how much pension they would get, because they would still be able to maintain a high anticipated utility after retirement. Also, there are other parameters such as one's happiness level, one's social life, and one's hobbies, which are hard to quantify but are important factors in determining the optimal retirement age for individuals in China. Therefore, future papers are expected to address the potential inadequacies and try to construct more solutions for people who are considering an optimal retirement age in China.

## Bibliography:

- [1]: Social Security History. (n.d.). Retrieved August 26, 2019, from <https://www.ssa.gov/history/age65.html>
- [2]: Labour Insurance Regulations of the People's Republic of China (1953 Amendment). (n.d.). Retrieved August 26, 2019, from <http://en.pkulaw.cn/display.aspx?cgid=96be87573ffd9605bdfb&lib=law>
- [3]: State Council Provisional Regulations on Retirement and Resignation of Workers. (n.d.). Retrieved August 26, 2019, from <http://www.chinalawedu.com/new/23223a23228a2010/20101216shangf14550.shtml>
- [4]: Life expectancy at birth, total (years) | Data. (n.d.). Retrieved from <https://data.worldbank.org/indicator/SP.DYN.LE00.IN?locations=CN>
- [5]: STATISTICAL COMMUNIQUE OF THE PEOPLE'S REPUBLIC OF CHINA NATIONAL ECONOMIC AND SOCIAL DEVELOPMENT. (n.d.). Retrieved August 26, 2019, from <http://r.cnki.net/knavi/yearbook/Detail/SSKY/YGMFZ?NaviID=&NO=>
- [6]: ZHAI, Z., CHEN, J., & Li, L. (n.d.). Aging in China: General Trends, New Characteristics and Corresponding Policies. Retrieved August 26, 2019, from <http://www.cnki.com.cn/Article/CJFDTOTAL-SDZS201603003.htm>
- [7]: Zhang, W., & Ren, Y. (2012). Construction and Application of Dynamic Model for Optimal Retirement Age under Change in Population Age Structure: A Case of Shaanxi Province. Northwest Publication, 33(1), 8.
- [8]: Pension Fund, Baidu Baike. (n.d.). Retrieved August 26, 2019, from <https://baike.baidu.com/item/%E5%85%BB%E8%80%81%E9%87%91/23517>
- [9]: The Decision on Major Issues Concerning Comprehensively Deepening Reforms at the close of the Third Plenary Session of the 18th CPC Central Committee. (n.d.). Retrieved August 26, 2019, from [http://language.chinadaily.com.cn/news/2013-11/18/content\\_17112855.htm](http://language.chinadaily.com.cn/news/2013-11/18/content_17112855.htm)
- [10]: Kenn B. Tacchino. 25 factors optimal retirement age[J]. Benefits Quarterly, 2013, 29(1): 39-47.
- [11]: Zhang, L., & Bian, J. (2015). Generalizations of the Methods of Measuring Optimal Retirement Age in China. Reform & Opening, (07). Retrieved from <http://www.cnki.com.cn/Article/CJFDTOTAL-GGKF201507037.htm>

- [12]: Lei, Y., & Pu, Y. (2004). Economics Modeling on Optimal Retirement Age Based on DB. *Journal of Industrial Technological Economics*, 23(1), 52–54. Retrieved from <https://www.ixueshu.com/document/88aa790a4e783877.html>
- [13]: Gao, Ji., & Li, P. (2017). Analyses of Decision to Delay Retirement Based on Individual Utility Maximization under the Background of Population Aging. *Population & Economics*, (3), 14–23.
- [14]: China's 1% National Sample Census in 2015. (n.d.). Retrieved August 26, 2019, from [http://www.stats.gov.cn/tjsj/zxfb/201604/t20160420\\_1346151.html](http://www.stats.gov.cn/tjsj/zxfb/201604/t20160420_1346151.html)
- [15]: Lei, X., Tan, L., & Zhao, Y. (2010). Does Retirement Affect Health. *China Economic Quarterly*, 9(4), 1540–1558.
- [16]: Li, J. (2015). The Effects of Longevity Risk on Decisions of Education and Retirement. *Statistical Research*, 32(2), 51–53.
- [17]: JAMES H, LANCE L, PETRA E T. Earnings functions, rates of return and treatment effects: the mincer equation and beyond [J]. *Handbook of the Economics of Education*, 2006 (1) : 317 - 329.
- [18]: Boss ZhiPin, R., & CSL. (n.d.). *Research On Gender Differences in China's Job Market*.

## **Acknowledgement:**

1. 这篇论文的选题来源于笔者长期对健康领域的关心及兴趣，再加上观察到周围的亲戚朋友和家人对退休生活的讨论及各自面临的退休问题所体现出的不同的生活状态，笔者想通过这篇论文来为中国社会找到适合每个人的最佳退休年龄。
2. 这篇文章笔者为唯一作者。
3. 指导老师高峰是笔者家人的朋友。高峰老师对笔者的这篇论文给予了无偿的大力支持和帮助。高峰老师从研究初期向笔者建议文献背景资料，到研究中期耐心的帮助笔者理解模型建构，再到研究后期给论文提出有力的修改意见，在此表示深深感谢！
4. 最后感谢丘成桐中学生科学奖给我这个机会，让我实现挑战自我并研究好奇和感兴趣的领域及问题，最终完成了有价值的成果。

本参赛团队声明所提交的论文是在指导老师指导下进行的研究工作和取得的  
研究成果。尽本团队所知，除了文中特别加以标注和致谢中所罗列的内  
容以外，论文中不包含其他人已经发表或撰写过的研究成果。若有不实之  
处，本人愿意承担一切相关责任。

参赛队员：Andrew Yang

指导老师：高峰

2019 09.01