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Chengyue Zhang

School: Phillips Exeter Academy

City, Country: Exeter, NH USA

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Inflation and Expectations: A Dual-Model Approach to Understanding U.S. Economic Dynamics

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Chengyue Zhang

Abstract:

The recent surge in U.S. inflation has ignited discussions on its implications for daily life and policy decisions. Amidst this, the intricate link between inflation and inflation expectations gains prominence. Inflation expectations not only influence actual inflation but also impact economic behavior. It is also a significant tool that the Federal Reserve uses to stabilize the economy. Our research bridges the gap by comprehensively analyzing inflation expectations' impact through Vector Autoregression (VAR) and Dynamic Stochastic General Equilibrium (DSGE) models. VAR captures short-term dynamics, while DSGE delves into microeconomic theory and policy effects. Using both of these research side-by-side offers insights into inflation expectations' influence on macroeconomic indicators. By enhancing our understanding, we provide valuable guidance for policy decisions, macroeconomic forecasts, and economic analyses in navigating the intricate economic landscape.

Keywords: Inflation expectation, VAR, DSGE, forecasting

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Commitments on Academic Honesty and Integrity

We hereby declare that we:

1. are fully committed to the principle of honesty, integrity and fair play throughout the competition.
2. actually perform the research work ourselves and thus truly understand the content of the work.
3. observe the common standard of academic integrity adopted by most journals and degree theses.
4. have declared all the assistance and contribution we have received from any personnel, agency, institution, etc. for the research work.
5. undertake to avoid getting in touch with assessment panel members in a way that may lead to direct or indirect conflict of interest.
6. undertake to avoid any interaction with assessment panel members that would undermine the neutrality of the panel member and fairness of the assessment process.
7. observe the safety regulations of the laboratory(ies) where we conduct the experiment(s), if applicable.
8. observe all rules and regulations of the competition.
9. agree that the decision of YHSA is final in all matters related to the competition.

We understand and agree that failure to honour the above commitments may lead to disqualification from the competition and/or removal of reward, if applicable; that any unethical deeds, if found, will be disclosed to the school principal of team member(s) and relevant parties if deemed necessary; and that the decision of YHSA is final and no appeal will be accepted.
(Signatures of full team below)

X Chengyue Zhang

Name of team member: Chengyue Zhang

X Susan Park

Name of supervising teacher: Susan Park

Date: 08/18/2023

1. Introduction

In recent years, the United States has witnessed a dramatic rise in inflation, a phenomenon that has significantly impacted people's daily lives and has become a central issue in political debates. According to the U.S. Bureau of Labor Statistics, general item prices have increased by 13% over the past two years. This surge has been particularly noticeable in essential goods such as groceries as well as in mortgage rates, leading to heightened consumer expectations for future inflation. During the severe inflationary period triggered by COVID-19, inflation expectations reached a peak of 6.9% in June 2022. In response, the U.S. government and the Federal Reserve have acted cautiously, implementing policies such as raising interest rates to mitigate both rising inflation and inflation expectations. In order to make the best decision in the overall economics, analysis of inflation and other related macroeconomic variables are brought into the spotlight (Ball et al., 2022; Crump et al., 2022; Del Negro et al., 2022).

As Ben Bernanke, the former head of the Federal Reserve, eloquently stated, “the state of inflation expectations greatly influences actual inflation and thus the central bank's ability to achieve price stability” (Bernanke, 2007). Inflation expectations have been found to not only shape economic behavior and decision-making but also directly impact the actual rate of inflation (Del Negro & Eusepi, 2009; Carlson & Parkin, 1975). When market participants, such as individuals or businesses, anticipate shifts in future prices, their actions can either exacerbate existing inflation or help mitigate inflationary pressures (Coibion et al., 2020a; Coibion et al., 2022). However, unlike other macroeconomic indicators, inflation expectation is a subjective concept, making it challenging to measure but also susceptible to manipulation (D'Acunto, 2023). Recognizing this, the U.S. central bank, the Federal Reserve, employs forward guidance as a policy tool to anchor inflation expectations, thereby maintaining price stability and sustainable economic growth (Bernanke, 2007; Coibion et al., 2020b). Beyond guiding central bank interest rates and monetary policy, inflation expectations also influence consumer spending, wage negotiations, and the labor market, further underscoring their importance (Chernov et al., 2012; D'Acunto, 2023).

Despite its pivotal role in shaping actual inflation, the importance of inflation expectations is often underestimated. As Janet L. Yellen, the chairwoman of the Federal Reserve in 2016, noted, “the precise manner in which expectations influence inflation deserves further study” (Yellen, 2016). Our research seeks to address this gap by contributing to the existing literature that explores the role of inflation expectation in shaping actual inflation and other macroeconomic indicators. By enhancing our collective understanding, we aim to assist policymakers, investors, and businesses in navigating the complex economic landscape more effectively.

Previous research has primarily investigated the relationship between inflation expectation and other macroeconomic indicators, such as interest rates and unemployment rates, using singular methods (Clark & Davig, 2011; Banbura et al., 2021; Mertens, 2016; Leduc et al., 2007). Our study, however, comprehensively analyzes the role that inflation expectation plays on other variables through two commonly used methods of macroeconomic modeling — Vector Autoregression (VAR) and Dynamic Stochastic General Equilibrium (DSGE). This dual approach allows for a more comprehensive and nuanced understanding.

Comparatively, VAR regresses each variable in the system on its own lagged values and the lagged values of all other variables, demonstrating the dynamics among multiple time series variables (Lütkepohl, 2013; Stock & Watson, 2001). It also reveals how the system responds to various short-term economic shocks through impulse response analysis. However, VAR's limitations in capturing complex nonlinear interactions and long-term relationships between variables necessitate the use of complementary methods (Giannone et al., 2014; Li et al., 2016). DSGE, constructed based on microeconomic theory and incorporating stochastic elements, analyzes economic dynamics and policy effects. It also captures the forward-looking behavior of economic agents, essential for understanding policy effects and economic dynamics, but requires intricate data to update its parameters (Del Negro et al., 2022). Overall, DSGE provides a more robust microeconomic theory, while VAR offers flexibility in analyzing high-dimensional time series data. DSGE is more commonly used for understanding long-run behavior, whereas VAR is used for short-term forecasting.

By utilizing both VAR and DSGE, our research leverages the strengths of each model to provide a clearer understanding of the relationship between inflation expectation and other macroeconomic indicators such as inflation, interest rate, and unemployment rate. This new insight into inflation expectations can form the basis for macroeconomic forecasting and offer valuable guidance for policymakers, investors, and economic analysts. Specifically, it enables policymakers to better utilize inflation expectations as a tool to counter undesired changes in inflation. Additionally, our comparative study between the two methods will further enrich the field by contrasting their separate results in the specific context of understanding inflation expectations, thereby providing a more holistic view of this critical economic phenomenon.

2. Related Work

In this section, we provide an overview of the existing literature on inflation expectations, focusing on the mechanisms that connect inflation and inflation expectations, and the various methods, particularly VAR and DSGE models, employed to analyze their role in the macroeconomic research.

2.1. Inflation Expectations in the United States

The relationship between inflation and inflation expectations is generally complex and interdependent. Consider the following scenario: When inflation is high, people's inflation expectations rise accordingly; the Federal Reserve then raises the interest rate to lower these expectations; the reduction in inflation expectations leads to a decrease in the actual inflation rate. This example illustrates the cyclical nature of inflation and expectations, where changes in one can affect the other. The Federal Reserve has long used inflation expectations to both monitor and influence inflation, evolving from early communication strategies in the 1990s to the contemporary practice of "forward guidance" to achieve general economic stability (Coibion et al., 2020a; Binder & Kamdar, 2022).

Despite its importance, measuring inflation expectations remains a contentious issue. Three generally discussed measurements include those of professional forecasters, firms, and consumers. Studies focusing on the U.S. economy often utilize data from 1) the Livingston Survey, a monthly survey from economists conducted by the Federal Reserve Bank of Philadelphia since 1946, 2) the Survey of Professional Forecasters (SPF), a quarterly survey conducted by the Federal Reserve Bank of Philadelphia since 1999, and 3) the Surveys of Consumers, a monthly survey conducted by the University of Michigan's Survey Research Center since 1946. These sources have been instrumental in shaping our understanding of inflation expectations, but they are not without controversy.

Researchers have debated whether there is a distinction between the inflation expectations of professional forecasters and those of households and firms, and which data is the most effective measurement (Mankiw et al., 2003). While professional forecasters may be more informed about recent inflation dynamics and monetary policy, many prefer the expectations of firms and consumers due to their more extended impact on the economy (Coibion et al., 2020a; Rudd, 2022). Firms' expectations influence critical decisions, including pricing, wage-setting, investment, and hiring, all of which heavily influence inflation (Bryan et al., 2015; Candia et al., 2021). Similarly, consumer expectations shape spending patterns, saving decisions, and investment choices (Kamdar, 2018; Reiche & Meyler, 2022).

The measurement of firms' and consumers' expectations requires careful consideration, as factors such as survey wording can influence reported expectations (Van der Klaauw et al., 2008; Armantier et al., 2013). Additionally, understanding the factors that influence these expectations requires deeper analysis (Andolfatto et al., 2008; Reiche & Meyler, 2022; D'Acunto et al., 2023). To better anchor these expectations, the central bank must communicate directly with the public, expanding beyond traditional Federal Open Market Committee (FOMC) statements and news media (Coibion et al., 2022). This highlights the need for more transparent and effective communication strategies to ensure that inflation expectations are well-anchored.

2.2. Vector Autoregressive (VAR) Models

VAR models have become a common and valuable approach to studying macroeconomic factors, such as inflation expectations. Various adaptations, such as Structural VAR, Bayesian VAR, and Global VAR, have been employed to investigate the role of inflation expectations in different economies. There is consensus that including inflation expectation data enhances general economic estimations (Hasenzagl et al., 2022; Rudd, 2022; Mertens; 2016), though disagreements persist about which type of expectation is most effective for improving forecasts.

Studies using long-term data in the U.S. have noted the decreasing volatility of long-term inflation expectations in the 21st century compared to the 1970s. Researchers have attributed this stability to the Federal Reserve's active role in adjusting monetary policy and anchoring long-term inflation expectations (Clark & David, 2011; Canova & Gambetti, 2010; Leduc et al., 2017; Mehra & Herrington, 2008). Using a medium-scale VAR model with time-varying parameters (TVP) and stochastic volatility, Clark & David (2011) conclude that the more stable long-term inflation expectation is related to more stable short-term inflation expectations and inflation itself. Canova & Gambetti (2010) investigate the role of expectations in the Great Moderation, a period of relatively stable economic conditions in the United States from the mid-1980s to the mid-2000s. Leduc et al. (2017) suggest that from 1979 to 2001 the Federal Reserve took a more active role in adjusting monetary policy in response to increasing expected inflation. Mehra & Herrington (2008) investigated how other macroeconomic variables influenced inflation from the 1970s and found that since the 1990s, the Federal Reserve’s anchoring of long-term inflation expectation allows fluctuation of short-term inflation expectation to have less impact on the long-term expectation or inflation itself. This has allowed short-term fluctuations to have less impact on long-term expectations or inflation itself, emphasizing the central role of the Federal Reserve's fiscal policy in the U.S. economy (Coibion et al., 2020).

Recent events, such as the dramatic rise of oil and gasoline prices in 2020 and 2021, have also prompted researchers to use VAR to investigate their relationship with inflation expectations (Aastvei, 2021; Kilian, 2022). Although changes in oil prices be related to changes in inflation expectations, we choose not to include this data because the consumer price index(CPI), which we already included in our research, already accounted for oil, one of the goods and services that represent the typical spending patterns of urban consumers. Moreover, global studies, such as those by Feldkircher & Siklos (2019) and Merten (2016), have further expanded the understanding of how changes in global inflation impact short-term inflation expectations, revealing variations over time and region. Instead, our study mainly focuses on inflation in the United States in more recent years that cover both the financial crisis and the COVID-19 pandemic periods.

2.3. Dynamic Stochastic General Equilibrium (DSGE) Models

The DSGE model has emerged as a pivotal framework for macroeconomic analysis and policy formulation, offering a more transparent system based on microeconomic theory (Christiano, 2018). Studies such as the Smets-Wouters model (Smets & Wouters, 2003) and the adoption of Bayesian techniques for parameter estimation (Lubik & Schorfheide, 2005; An & Schorfheide, 2007; Smets & Wouters, 2007) have significantly advanced the field. For example, later known as the Smets-Wouters model or SW model, Smets & Wouters (2003) effectively replicated crucial aspects of economic fluctuations, commonly referred to as business cycles (Smets & Wouters, 2003). Concurrently, in 2005, Lubik and Schorfheide

(2005) adopted Bayesian techniques for parameter estimation of the DSGE model. These developments have allowed the DSGE model to better replicate crucial aspects of economic fluctuations and fit real-world data.

The application of DSGE models holds substantial significance in central banking for policy analysis and forecasting (Tovar, 2009). However, the robustness of the model has been questioned, particularly during intense monetary fluctuations (Edge, 2010). The integration of inflation expectation data has emerged as a potential solution to enhance forecasting accuracy.

Researchers have increasingly recognized the importance of utilizing inflation expectations data within DSGE models to evaluate their capacity to capture complex economic dynamics. This approach has become particularly relevant during periods of substantial disruptions, such as financial crises or significant policy shifts (Del Negro & Eusepi, 2011; Doser et al., 2017; Milani, 2023). Within the framework of dynamic equilibrium, all variables within the system interact to maintain balance. Minor fluctuations in one variable prompt adjustments throughout the system, ensuring a new equilibrium is reached. However, significant external shocks pose a challenge in identifying the subsequent equilibrium point. In this context, the incorporation of inflation expectations becomes crucial. Del Negro & Eusepi (2011) specifically highlighted the use of an inflation expectation augmented linear Phillips curve as a promising avenue for refining the DSGE model's ability to navigate these complexities.

The integration of inflation expectations into DSGE models is not a straightforward task, and different methodologies have been proposed. In 2019, Gelain et al. expanded on this concept by comparing the effect of using hybrid expectations, which combine both rational and adaptive expectations, with the traditional rational expectations within the DSGE framework (Gelain et al., 2019). This comparison revealed nuanced differences in how these models respond to economic shocks, providing insights into the underlying mechanisms that drive inflation expectations. Similarly, Slobodyan & Wouters (2012) and Warne (2023) conducted comparative studies to analyze the forecast results of the DSGE model with rational expectations and the adaptive learning model. These studies contribute to the ongoing debate on the most effective ways to model expectations within the DSGE framework, recognizing that different approaches may be more suitable depending on the specific economic conditions and policy questions at hand.

These advancements in DSGE modeling, coupled with the integration of inflation expectations, represent a promising direction for future research. They offer the potential to deepen our understanding of the dynamic interplay between inflation expectations and other macroeconomic variables, providing valuable insights for policymakers and economists alike.

3. Data

3.1. Data Overview

In our research, we employ four key time-series data sets: inflation expectation, inflation, interest rate, and unemployment rate, respectively. These data sets are calculated monthly, and our analysis spans from January 2000 to April 2023 which may cover both the stable periods and market fluctuations during the 2009 financial crisis as well as the COVID-19 pandemic periods.

3.1.1. Inflation Expectation Data

For inflation expectation, we utilize data from The Surveys of Consumers conducted by the Survey Research Center at the University of Michigan (http://www.sca.isr.umich.edu/). Specifically, we used the monthly median one-year ahead expected inflation rate column for the aforementioned period. This data is collected through a survey of 500 households and represents one of the most long-standing and reputable sources for consumers' inflation expectations. The utilization of this data is crucial in understanding how consumers perceive future price changes, which can significantly influence their spending and saving behaviors (University of Michigan).

3.1.2 Inflation Rate Data

Inflation data is derived from the Consumer Price Index (CPI) provided by the U.S. Bureau of Labor Statistics (https://www.bls.gov/cpi/data.htm). The CPI is a widely used economic indicator that measures the average change over time in the prices paid by urban consumers for a broad array of goods and services. It reflects the general cost of living and inflation for consumers, offering a comprehensive view of price stability within the economy.

3.1.3. Interest Rate Data

The federal funds rate, representing the interest rate, is retrieved from the FRED dataset provided by the Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/FEDFUNDS). This rate, at which banks lend reserve balances to each other overnight, serves as a critical measurement of the Federal Reserve's monetary policy. By altering the federal funds rate, the Fed can influence borrowing costs and credit availability in the economy. Lowering the rate may stimulate economic growth and employment by encouraging borrowing, spending, and investment. Conversely, during periods of rising inflation, the Federal Reserve may raise interest rates to curb excessive economic growth, making this data essential for understanding monetary policy's impact on the economy.

3.1.4. Unemployment Rate Data

The unemployment rate data is also sourced from the FRED dataset by the Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/UNRATE). The unemployment rate offers valuable insights into an economy's health and labor market conditions. Policymakers often rely on this data to formulate measures for promoting economic growth, job creation, and stability. Understanding the unemployment rate is vital for guiding economic policies and assessing the overall state of the economy.

3.1.5. GDP Growth Data

In the DSGE model, we also incorporate the Monthly GDP (MGDP) Index data from 1992 to June 2023 (https://www.spglobal.com/marketintelligence/en/mi/products/us-monthly-gdp-index.html). This monthly data is calculated and aggregated using methods akin to those employed by the official U.S. Bureau of Economic Analysis for quarterly GDP. This approach ensures that the resultant monthly index accurately replicates the fluctuations observed in official quarterly GDP, while simultaneously capturing insightful representations of monthly shifts within each quarter.

3.2. Explorative Data Analysis

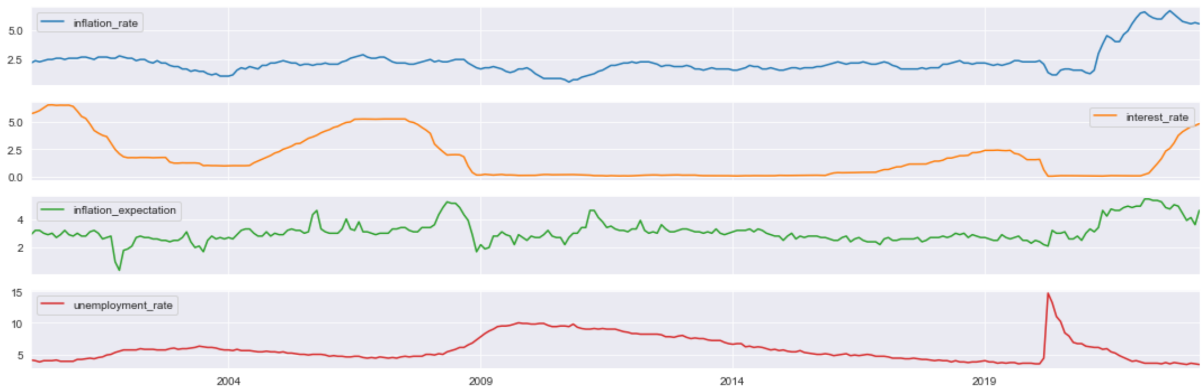
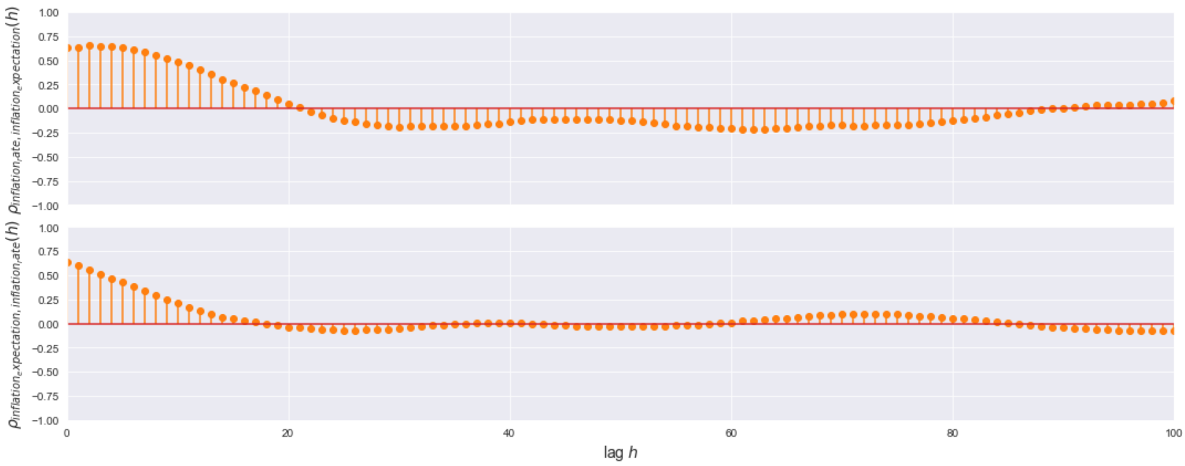


Figure 1. Monthly data of inflation expectation, inflation, interest rate, and the unemployment rate for analysis from January 2000 to April 2023.

Figure 1 shows the four main data that we used in our research — inflation expectation, inflation, interest rate, and the unemployment rate. As we can see the inflation rate and inflation expectation of the past few years have been record high since the Great Moderation period. The unemployment rate also hit a peak after 2019, and the interest rate has been increasing as one of the Federal Reserve's efforts to manage inflation and inflation expectations.

Prior to commencing our research, we conduct exploratory data analysis employing Cross-Correlation Functions (CCF) to statistically measure whether the multivariate approach offers advantages over treating the signals individually as univariate time series. CCF uses statistical techniques to indicate the degree of similarity between two time-series datasets. From our analysis (Figure 2), we observe that the inflation rate and inflation expectation have a positive correlation, inflation and interest rate are inversely related, inflation expectations and interest rate are also inversely related, and inflation rate and unemployment rate exhibit similar trends. This preliminary analysis helps in understanding the underlying relationships between the variables and informs the modeling strategies we adopt in our research.



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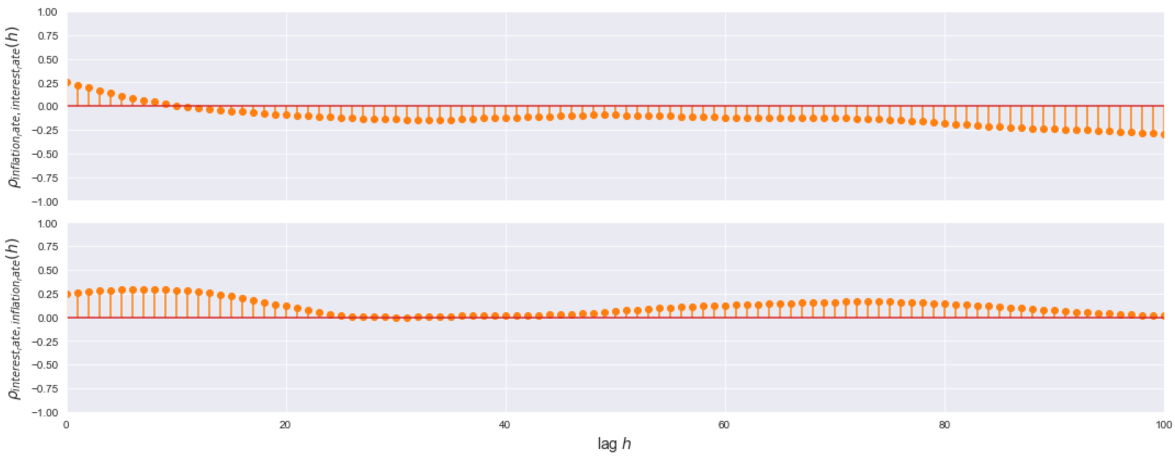

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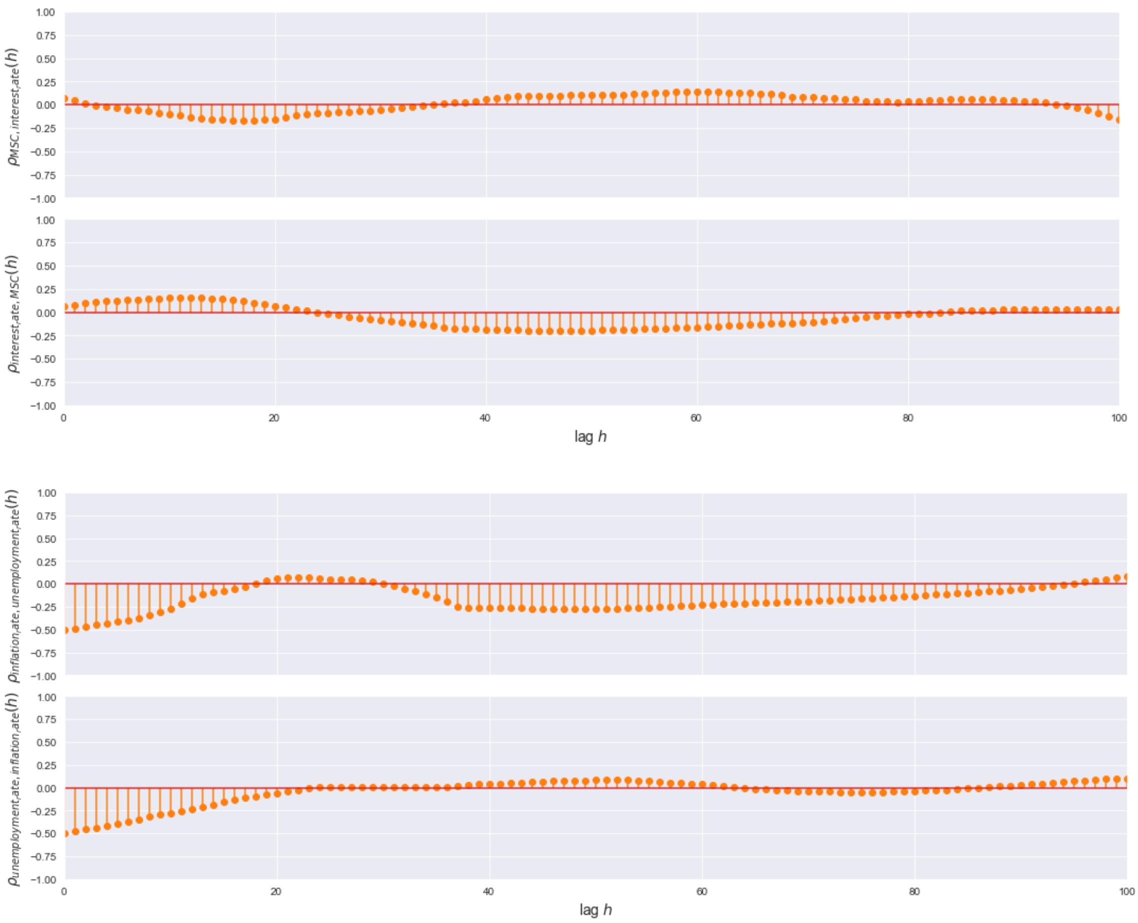


Figure 2. Pairwise cross-correlation functions (CCF) between the four time series

4. Method

4.1. Vector Autoregressive Method (VAR)

The Vector Autoregressive (VAR) method is a common statistical approach used to analyze time-series data. In this section, we employ the VAR method to model the dynamics of the U.S. economy, focusing on four key variables: inflation rate, interest rate, inflation expectation, and unemployment rate.

4.1.1. Model Specification

We first assume that the dynamics of the U.S. economy can be modeled by the following structural VAR model:

$$Y_t = a_0 + \sum_{j=1}^{p} A_j y_{t-j} + u_t \quad (1)$$

Where Y_t is a 4x1 data vector of four endogenous variables at time t, containing monthly data for the inflation rate, interest rate, inflation expectation, and unemployment rate. a_0 is the 4x1 vector of intercepts, A_j is the 4x4 matrix of coefficients for the j-th lags of the endogenous variables collected in y_{t-j} , and u_t is the 4x4 covariance matrix of the VAR disturbance. P is the number of lags included in the variable to be determined later in the process.

The general equation for a structural VAR model can be exemplified as a bivariate system, where $y_{1,t}$ and $y_{2,t}$ are both determined by lagged values of $y_{1,t-1}$ and $y_{2,t-1}$. $A_{i,j}$ are coefficients that represent how the past values of y_j influence the current y_i value at time t:

$$y_{1,t} = a_{0,1} + A_{1,1} \times y_{1,t-1} + A_{1,2} \times y_{2,t-1} + u_{1,t} \quad (2)$$

$$y_{2,t} = a_{0,2} + A_{2,1} \times y_{1,t-1} + A_{2,2} \times y_{2,t-1} + u_{2,t} \quad (3)$$

Writing the system in matrix form, the system becomes:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} a_{0,1} \\ a_{0,2} \end{bmatrix} + \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \quad (4)$$

Then let

$$Y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix}; a_0 = \begin{bmatrix} a_{0,1} \\ a_{0,2} \end{bmatrix}; A = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix}; u_t = \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \quad (5)$$

the bivariate system can be written as follows:

$$Y_t = a_0 + A \times y_{t-1} + u_t \quad (6)$$

Adding in more possible p-order, Y_t become related to all past values before time p instead only values at time t-1. Then we obtain the general equation for a structural VAR model.

4.1.2. Stationarity Tests

To assess stationarity, we employ the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test and the Augmented Dickey-Fuller (ADF) test stationarity, respectively.

	inflation_rate	interest_rate	inflation_expectation	unemployment_rate
Test statistic	0.2933	0.2128	0.1616	0.3134
p-value	**0.0100**	**0.0112**	**0.0370**	**0.0100**
Critical value - 1%	0.2160	0.2160	0.2160	0.2160
Critical value - 5%	0.1460	0.1460	0.1460	0.1460
Critical value - 10%	0.1190	0.1190	0.1190	0.1190

Table 1. Result from KPSS Test (small p-values are bolded)

	inflation_rate	interest_rate	inflation_expectation	unemployment_rate
Test statistic	-0.4359	-3.6232	-3.1371	-2.8331
p-value	0.9039	**0.0053**	**0.0239**	0.0537
Critical value - 1%	-3.4552	-3.4545	-3.4544	-3.4541
Critical value - 5%	-2.8725	-2.8722	-2.8721	-2.8720
Critical value - 10%	-2.5726	-2.5724	-2.5724	-2.5723

Table 2. Result from ADF Test (small p-values are bolded)

The null hypothesis of the KPSS test is that a given time series is stationary around a deterministic trend (Kwiatkowski et al., 1992). By comparing the test statistic with the critical values from the KPSS distribution, we find that the test statistic is generally bigger than the critical values - 5% (relatively small p-value), suggesting the null hypothesis is not rejected. Thus, the time series is considered not stationary.

The null hypothesis of the ADF test is that the time series has a unit root and is non-stationary (Cheung & Lai, 1995). The test statistic of the inflation rate and inflation expectation is less negative (or bigger) than the critical values (relatively big p-value), suggesting the null hypothesis is not rejected. Conversely, the test statistic of the unemployment rate and interest is more negative (or smaller) than the critical values (relatively small p-value), suggesting the null hypothesis is rejected. Thus, the inflation rate and inflation expectation time series are not stationary, while the unemployment rate and interest time series are stationary.

4.1.3. Data Transformation through Differencing

To transform our data to be stationary, we take the first-order difference of all four data series. This operation computes the difference between the current value and the previous value in the time series. The resulting series still captures the changes in time series data while exhibiting less trend and becoming more stationary. Let it be the original time series data at time t. The first-order difference is calculated as:

$$z_t = y_t - y_{t-1} \quad (7)$$

The resulting series still captures the changes in time series data while exhibiting less trend and becoming more stationary. After taking the first-order differences, we tested for stationarity again, and all four-time series satisfied both KPSS and ADF tests (Tables 3 and 4, respectively). As shown in Table 3, the p-value of all four data are big, meaning that the KPSS test's null hypothesis is not rejected and the data is stationary. As shown in Table 4, the p-values of all four data are small, meaning that the ADF test's null hypothesis is rejected and the data is now stationary.

	inflation_rate	**interest_rate**	**inflation_expectation**	**unemployment_rate**
Test statistic	0.065	0.0434	0.018	0.031
p-value	0.1000	0.1000	0.1000	0.1000
Critical value - 1%	0.2160	0.2160	0.2160	0.2160
Critical value - 5%	0.1460	0.1460	0.1460	0.1460
Critical value - 10%	0.1190	0.1190	0.1190	0.1190

Table 3. Result from KPSS Test After First-Order Differencing

	inflation_rate	**interest_rate**	**inflation_expectation**	**unemployment_rate**
Test statistic	-5.7443	-8.9097	-8.6139	-12.9020
p-value	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Critical value - 1%	-3.4552	-3.4545	-3.4544	-3.4541
Critical value - 5%	-2.8725	-2.8722	-2.8721	-2.8720
Critical value - 10%	-2.5726	-2.5724	-2.5724	-2.5723

Table 4. Result from ADF Test After First-Order Differencing (small p-values are bolded)

Then, we split the data into train and test sets. We use the dataset from 2000 January to 2022 October to predict the last six months in the dataset (2022 November to 2023 April).
Next, we determine the model order or lag length, denoted as p, by calculating four commonly used multivariate information criteria: Akaike Information Criterion (AIC), Schwarz's Bayesian Information Criterion (BIC), the Hannan-Quinn Criterion (HQ), and Akaike's Final Prediction Error Criterion (FPE).

Figure 3. Results of multivariate information criteria, AIC, BIC, HQIC, and FPE

We select the set of order parameters associated with the lowest values. As seen in Figure 3, we find BIC and HQIC to be lowest at p=2, and we also observe an elbow in the plots for AIC and FPE, so we choose the number of lags to be 2. After selecting the p-order, we fit the VAR model with the chosen order and obtain a forecast. This process allows us to analyze the relationships between the variables and make predictions about future trends, providing valuable insights into the dynamics of the U.S. economy.

4.2. DSGE (Dynamic Stochastic General Equilibrium) Model

4.2.1. Model Overview

In our study, we follow the basic Smets-Wouters (SW) DSGE model, as described in Smets and Wouters (2007), with the addition of inflation expectation as defined by Del Negro & Schorfheide (2013). The SW model is a prominent New Keynesian model that captures the macroeconomic dynamics of the economy using seven different parameters (endogenous variables), three of which (GDP, inflation, interest rate) are also exogenous shocks.

4.2.2. Model Equations

The log-linearized equilibrium conditions of the SW model introduce long-run growth through a total factor productivity process, and the model is detrended to express almost all equilibrium conditions in a way that encompasses both trend-stationary and unit root processes for technology.

Formally, here are the given equations:

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma}[\hat{i}_t - E_t(\pi_{t+1})] + \psi^n_{ya}(\rho_a - 1)a_t \quad (8)$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa\tilde{y}_t + \sigma_\pi \varepsilon^\pi_t \quad (9)$$

$$\hat{i}_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \quad (10)$$

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon^a_t \quad (11)$$

$$v_t = \rho_v v_{t-1} + \sigma_v \varepsilon^v_t \quad (12)$$

In these equations, y_t is an endogenous variable representing the output gap at time t; π_t represents the inflation at time t; i_t represents the interest rate at time t. where the following parameters are given by:

$$\psi^n_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \quad (13)$$

$$\kappa = \frac{(1-\theta)1-\theta\beta[\sigma(1-\alpha)+\varphi+\alpha}{\theta(1-\alpha+\alpha\epsilon)} \quad (14)$$

In the model, the expectations of time t+1 variables are expressed as time-t variables, and the relationship between variable expectations and the effective value are captured by the expectational error. The equations are given by:

$$\tilde{y}_{t+1} = E_t(\tilde{y}_{t+1}) + \eta^y_{t+1} \quad (15)$$

$$\tilde{\pi}_{t+1} = E_t(\tilde{\pi}_{t+1}) + \eta^y_{t+1} \quad (16)$$

To account for the role of inflation expectation on inflation, equation (20) defines the relationship between inflation and inflation expectation t as the expected inflation at given time t+1.

Thus, the model consists of 7 endogenous variables, 3 exogenous shocks ε, and 2 expectational errors η that compose the state equations of the model.

The observation equations specify the relationship between the underlying state variables and the observable variables. In our model, we assume that only the output gap, inflation, interest rate, and inflation expectation are observable variables. The observation equations are:

$$GDP_t = \tilde{y}_t \quad (17)$$

$$inflation_t = \pi_t \quad (18)$$

$$interest\_rate_t = \left(\frac{1}{\beta} - 1\right) + \hat{i}_t \quad (19)$$

$$inflation\_expectation_t = E_t(\pi_{t+1}) = (\pi_t + \kappa y_t + \tilde{\sigma}_\pi)/\beta \quad (20)$$

These equations link the theoretical constructs of the model to the data that can be empirically observed, allowing for estimation and validation of the model.

4.2.3. Model Calibration and Estimation

The estimation of DSGEs often encounter the challenge of parameter identification. To address this issue, parameters that are not identified are typically calibrated prior to estimation. In particular, we calibrate the parameters by establishing a dictionary containing only the parameters we intend to calibrate and then construct a matrix for the parameters that will be estimated.

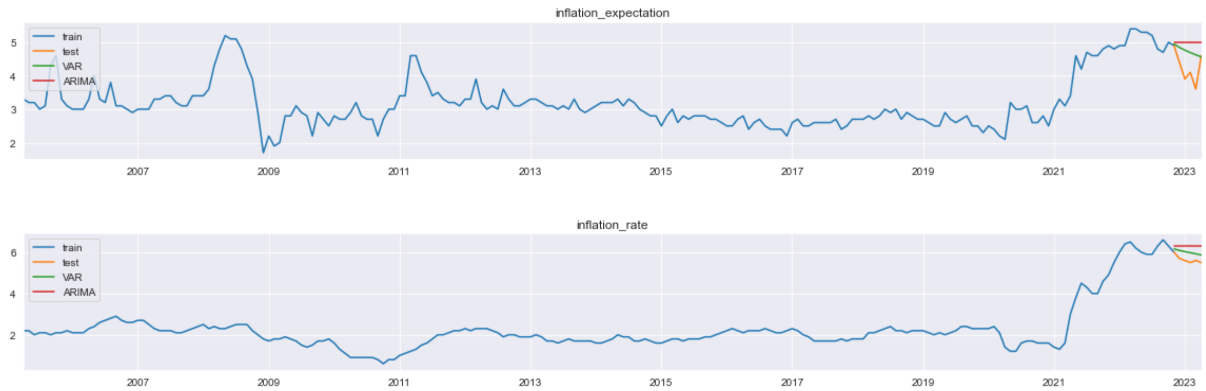
Since we employ Bayesian estimation with MCMC (Herbst and Schorfheide, 2014), it is necessary to define the priors for the parameters to be estimated. Various distributions are available for priors, including Beta, Gamma, Inverse Gamma, Uniform, and Normal. Each prior requires the specification of a mean and a standard deviation, and the relationship between these statistics and the distribution parameters is outlined in Table 5 below.

Parameter	Distribution	Mean	Standard Deviation	Label
σ	Normal	1.30	0.20	σ
θ	Beta	0.60	0.20	θ
ϕ_{p_i}	Normal	1.50	0.35	ϕ_{p_i}
ϕ_Y	Gamma	0.25	0.10	ϕ_Y
ρ_a	Beta	0.50	0.25	ρ_a
σ_a	InvGamma	0.50	0.25	σ_a
ρ_v	Beta	0.50	0.25	ρ_v
σ_v	InvGamma	0.50	0.25	σ_v
σ_{p_i}	InvGamma	0.50	0.25	σ_{p_i}

Table 5. Prior distributions, means, and standard deviations for each estimated parameter

5. Result

5.1. VAR Result



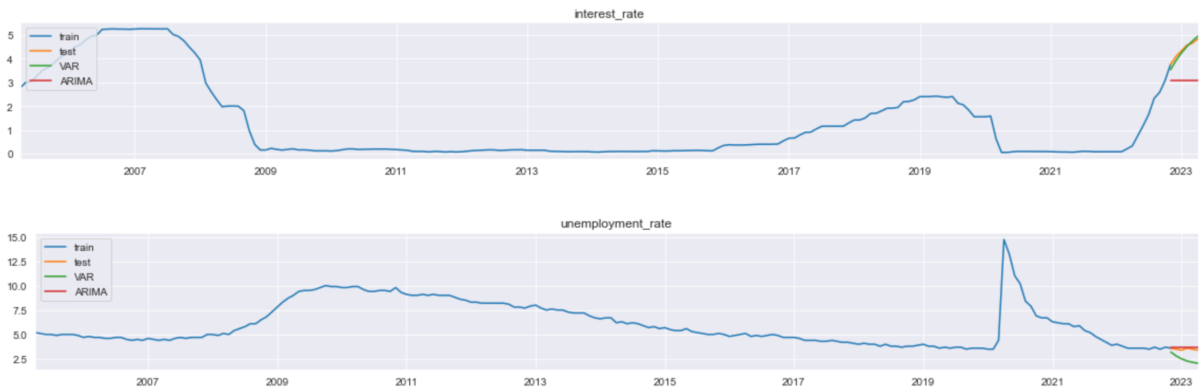


Figure 4. Estimation of the VAR model in comparison to ARIMA

In Figure 4, we can see that the forecast of our VAR model shown in orange is closer to the actual trend than to the Autoregressive Integrated Moving Average (ARIMA) forecast, which only models a single time series variable and its relationship with its past values, in inflation expectation, inflation rate, and interest rate. The two methods obtain similar results in the unemployment rate. Additionally, our VAR model can forecast the general trend of variables except the unemployment rate. A more numerical comparison between the VAR and ARIMA models can be seen in Table 3.

As seen in Table 6, in all three different potential matrices — Mean Absolute Error (MAE), Mean Squared Error (MSE), and Mean Absolute Percentage Error (MAPE) — VAR forecasts have lower errors than ARIMA forecasts for the inflation rate, interest rate, and inflation expectation, but a higher error in the unemployment rate. (The lower error is bolded in the table for clearer visualization). This proves the robustness of our VAR model to be better than the results of the ARIMA model.

		inflation_rate	interest_rate	inflation_expectation	unemployment_rate
MAE	VAR	**0.356953**	**0.123469**	**0.501983**	0.995051
	ARIMA	0.650000	1.290869	0.750000	**0.200000**
MSE	VAR	**0.137422**	**0.018312**	**0.391682**	1.123061
	ARIMA	0.451667	1.797508	0.751667	**0.046667**
MAPE	VAR	**6.368140**	**2.939176**	**12.830437**	28.517707

	ARIMA	11.602871	29.024414	18.903011	**5.771864**

Table 6. Performance Metrics Comparing ARIME and VAR Results

Next, in Figures 5 and 6, we examine the two methods in summarizing the effects causal impacts that the VAR model provides — Impulse Response Functions (IRFs) and Forecast Error Variance Decomposition (FEVDs). IRFs show how a shock to one variable affects the behavior of all other variables in the system over time. FEVDs indicate the contribution of each variable in variations in forecast errors in the future. These two functions of the VAR models allow us to analyze the interconnected relationship between the four macroeconomic variables.

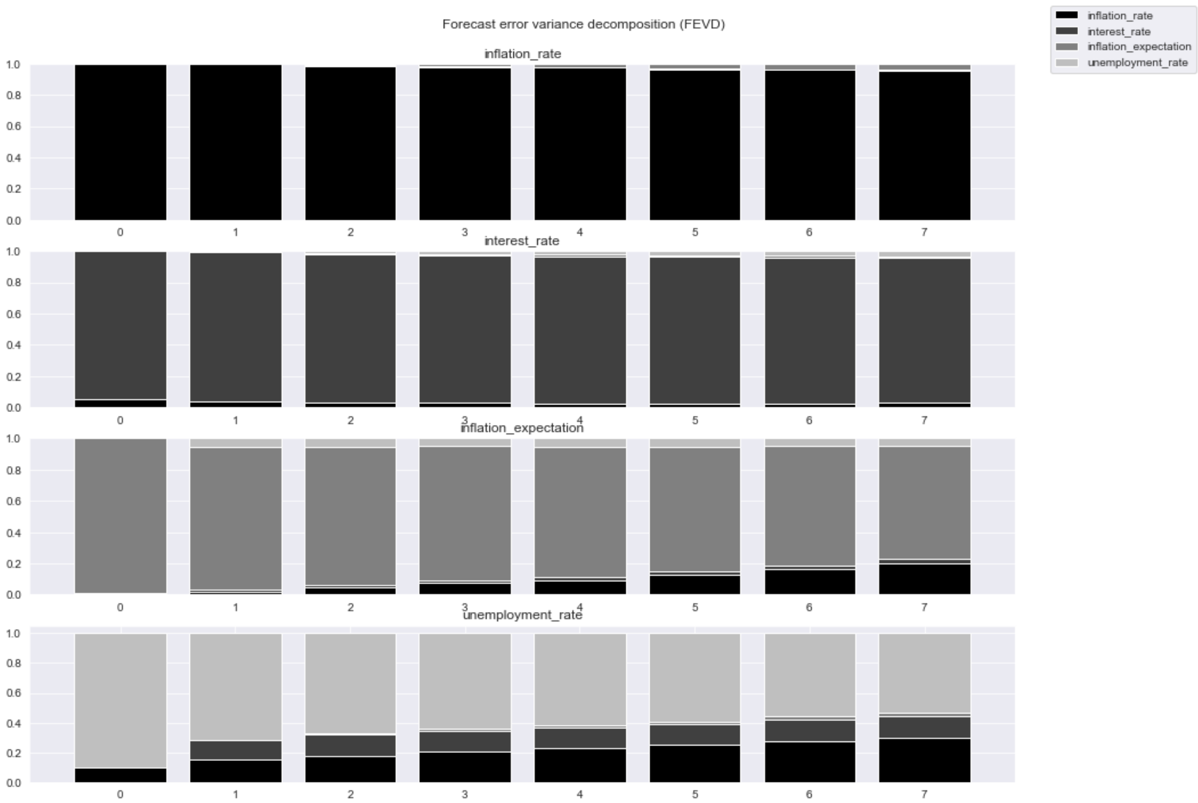


Figure 5. Forecast Error Variance Decompositions (FEVDs) of the VAR model

From Figure 5, we can see that variance in inflation rate and interest rate primarily influences its own future values with little impact on other variables. To be more specific, variance in the inflation rate has a small influence on inflation expectation in later periods, and variance in the interest rate has a small influence on the inflation rate during initial periods and a small increasing influence on the unemployment rate. Variance in inflation expectation, however, has a comparatively more significant and increasing influence inflation rate over time. Besides, it also has a noticeable influence on the unemployment rate. Variance in the unemployment rate has the most influence on other variables, with a gradually increasing impact on the inflation rate and a relatively consistent but significant impact on inflation expectation.

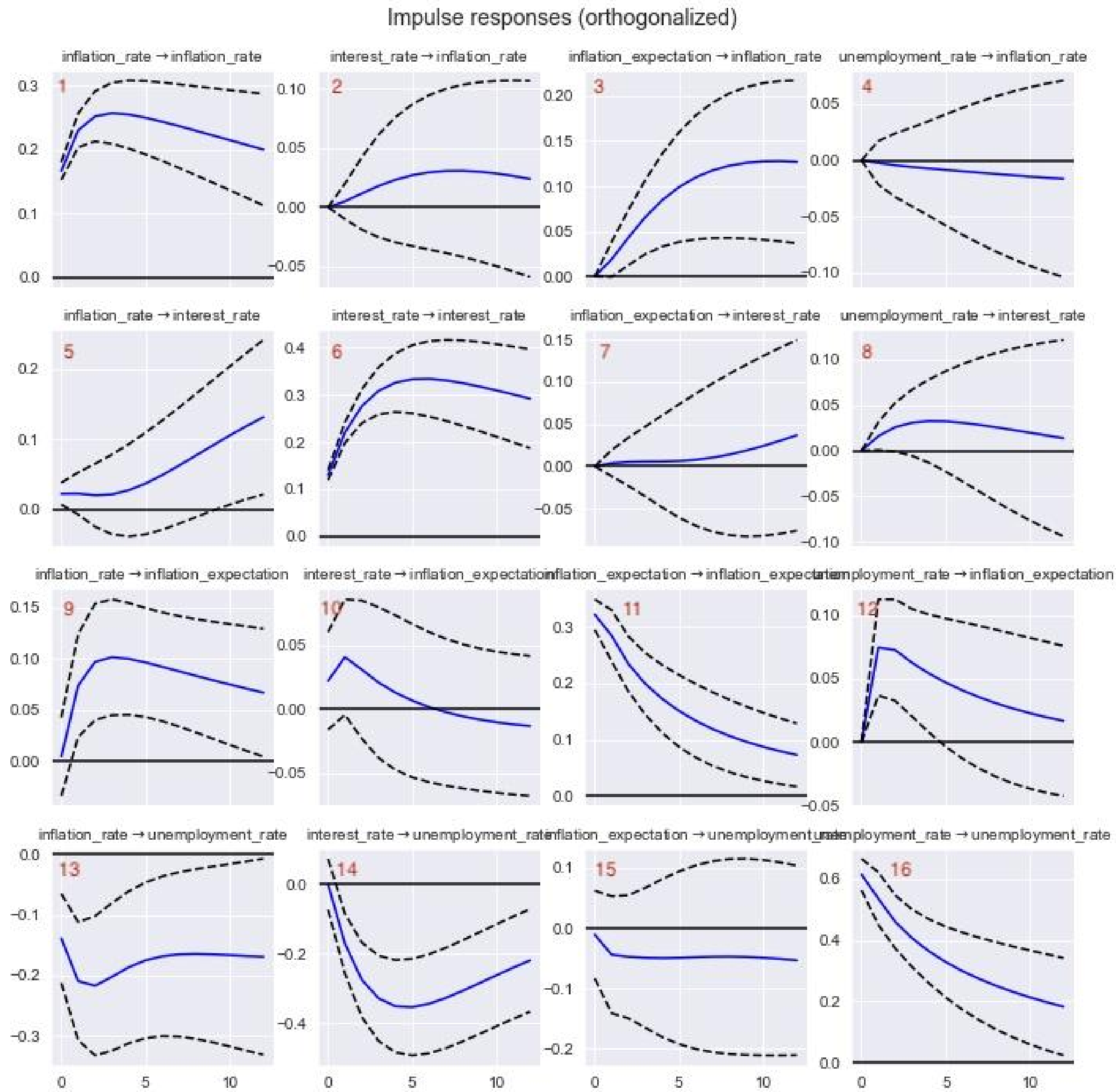


Figure 6. Impulse Response Functions (IRFs) of the VAR model

All sub-graphs in Figure 6 are labeled by numbers. First of all, the four graphs (1, 6, 11, 16) top-left to bottom-right diagonal line shows the four variables influence on itself. All of them have a significant positive influence on itself, with inflation expectations and the unemployment rate's impact decreasing faster. In sub-graph 16, the unemployment rate has an especially high magnitude (0.6) of influence on itself. Looking across the sixteen sub-graphs, in sub-graphs 3 and 9, we can see the inflation rate and inflation expectation both have a strong correlation to each other with up to a quantitative number of 0.10.

Then going by order, in sub-graphs 2-3, interest rate and inflation expectation have increasing positive influences on the inflation rate, with inflation expectation being the more significant. In sub-graph 4, we can see that the unemployment rate has a slight negative influence on the inflation rate. In the second row, all three other variables have a positive influence on the interest rate. The inflation rate's influence is the most significant on the interest rate, and the inflation rate and inflation expectation's influence are similar in trend. This response corresponds to our understanding that inflation expectation dictates inflation and the Federal Reserve may increase the interest rate in an attempt to anchor inflation expectation. In the third row, all three other variables have initially positive influences on inflation expectations but their influences all start decreasing over time after around two to three periods. The inflation rate has the most significant influence on inflation expectation besides inflation expectation itself. In the fourth row, all three variables have a negative influence on the unemployment rate.

5.2. DSGE Result

Figure 7. The Fit and Forecast Result of the DSGE Model

In Figure 7, the blue line represents the actual historical data, while the orange line represents the forecast of the DSGE model. In the case of GDP and interest rate, our model provides a fairly accurate forecast, with the forecasted value almost exactly overlapping the actual value. However, DSGE model performs relatively less accurately for inflation rate and inflation expectation especially during the COVID-19 period, indicating its limitation under the strong market fluctuation (Edge et al. 2010). In particular, it would underestimate the inflation rate in post-COVID19 era, while overestimating the inflation expectation as the response to the actual inflation.

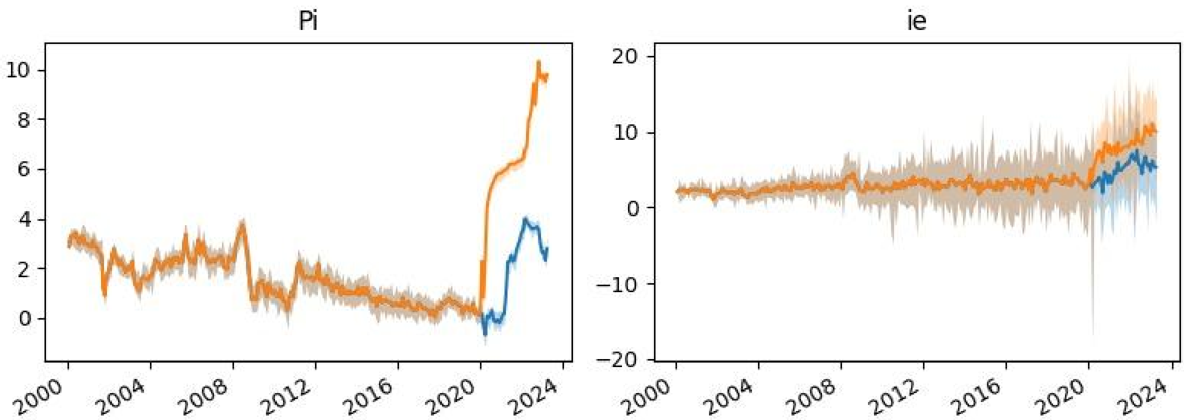


Figure 8. Counterfactual analysis of Inflation Rate and Inflation Expectations to exogenous shock of zero interest rate

Furthermore, we intend to show how the inflation expectation may help change the inflation rate using a counterfactual analysis approach, where we set the interest rate to be 0 after year 2020, i.e., the pandemic period. Figure 8 shows that both the hypothetical inflation rate and inflation expectation would rise significantly over 10% by 2024, had the Federal Reserve choose not to adjust the interest rate. As such, both the inflation rate and inflation expectation would respond to the exogenous shock with strong similarity

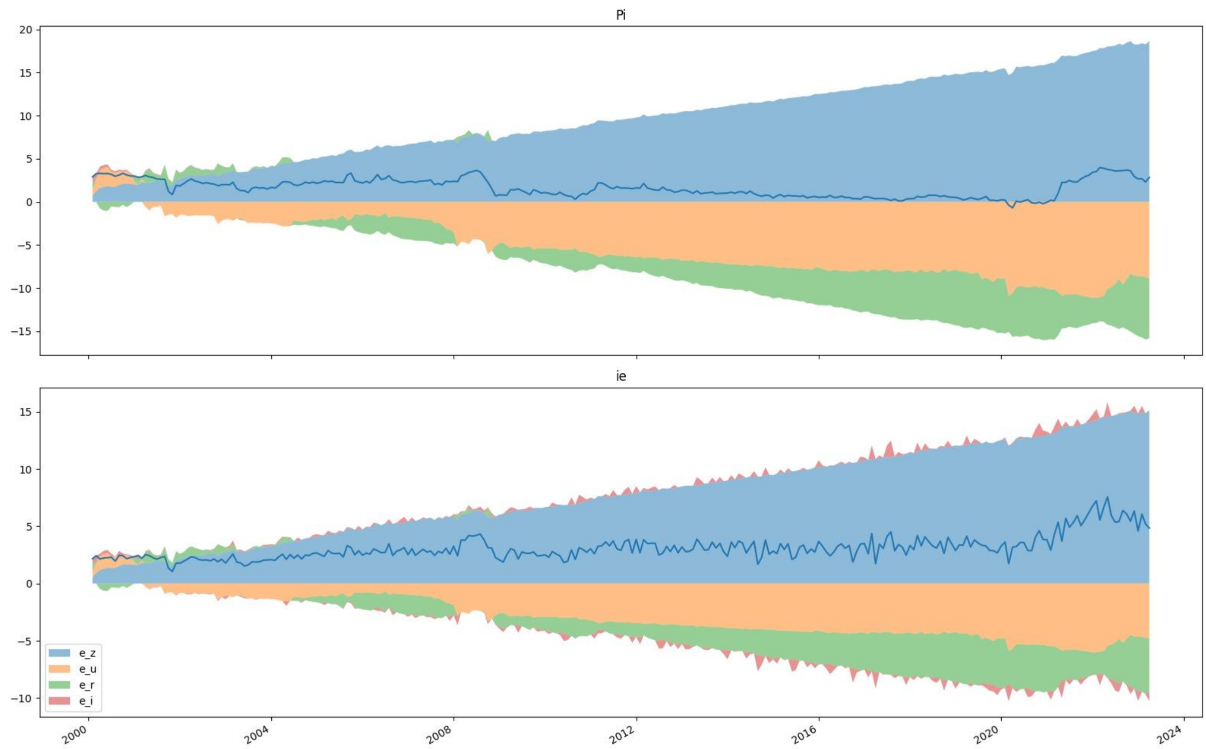


Figure 9. Historical Shock Decomposition of Observable Variables

Lastly, we perform the historical shock decomposition on both inflation and inflation expectation to evaluate the role of historical structural shocks to the observables. More specifically, the Kalman filter is employed to estimate the historical structural shocks and other unobservable model variables given the observables. We find that inflation rate is mostly driven by the known shocks, while inflation expectation would also be driven by the shock originating from itself.

6. Discussion

A fundamental distinction between Vector Autoregression (VAR) and Dynamic Stochastic General Equilibrium (DSGE) models lies in the level of structural constraints they impose. DSGE models, characterized by a more comprehensive structural framework, entail a greater number of assumptions and restrictions on the relationships among variables (Christiano, 2018; Del Negro et al., 2022). In contrast, VAR models offer a more flexible approach with fewer predefined constraints, although they require preliminary tests such as stationary tests and p-value assessments to determine the appropriate model specifications for estimation (Christiano et al., 2005; Giacomini 2013).

In our study, we utilize inflation expectation data sourced from the esteemed University of Michigan, which primarily captures consumer or household inflation expectations. However, it is important to acknowledge that inflation expectations among firms and professional forecasters exhibit distinct patterns compared to those of households (Coibion et al., 2022b). Thus, a limitation of our research stems from the inherent divergence between these categories of inflation expectations. Moreover, the distinction between long-term and short-term inflation expectations also needs to be considered when used to represent general inflation expectations (Moessner & Takáts, 2020; Nautz & Strohsal, 2015).

A notable constraint of the VAR model is its inherent backward-looking nature. This attribute underscores the model's reliance on historical events, which may not fully mirror the current economic landscape. In contrast, the DSGE model stands as a structural methodology, anchoring its foundation in theoretical constructs where all variables are meticulously defined as equations, thus enabling a more comprehensive representation of economic dynamics (Christiano et al., 2005; Mehra & Herrington, 2008).

In essence, the comparison between VAR and DSGE encapsulates the dichotomy between the reduced-form historical analysis of VAR and the structurally-grounded theoretical framework of DSGE. Each approach offers unique insights into economic phenomena, with VAR's flexibility complementing DSGE's precision and theoretical underpinning.

7. Conclusion

7.1. Implication

This study delved into the intricate relationship between inflation and inflation expectations through the utilization of two distinct methodologies: VAR and DSGE models. The results of both approaches yielded a pronounced correlation between these interconnected variables. Furthermore, we conducted a methodical comparison between the VAR and DSGE models, showcasing the alignment and disparities in their outcomes. By juxtaposing the reduced-form analysis of VAR with the structured theoretical framework of DSGE, we unveiled valuable insights into the convergence of their findings.

In light of these findings, we advocate for policymakers to take into account inflation expectations as a means to engender economic stability and control inflationary pressures. To better manage inflation expectations, policymakers can adopt better strategies. Enhancing transparency in policy decisions, such as disseminating meeting notes and fostering clear communication, can enhance the credibility of central bank actions. Additionally, recognizing the existence of a symbiotic relationship between expectations and economic activities prompts a comprehensive exploration, extending beyond consumer expectations to incorporate production and supply side (firm) expectations. By encompassing a holistic understanding of these multifaceted dynamics, policymakers can forge a more nuanced and effective approach to inflation management.

7.2. Limitation and Future Work

While this research has contributed valuable insights into the intricate relationship between inflation and inflation expectations, several limitations warrant consideration and offer avenues for future exploration.

Our analysis relies on a dataset comprising four macroeconomic variables, including the Consumer Price Index (CPI) as a measure of overall inflation. It is important to acknowledge that the choice of CPI as the inflation metric may not encapsulate all facets of inflation dynamics (Reed & Rippy, 2012). Future research could investigate the suitability of alternative inflation measures, considering potential variations in index calculation across different countries. Moreover, expanding the dataset to include a broader array of macroeconomic indicators could enhance the comprehensiveness of our findings.

This study employed established methodologies, namely VAR and DSGE models, to explore the interplay between inflation and inflation expectations. However, the inherent limitations of these methods and their assumptions prompt the consideration of innovative approaches for future research. One promising avenue is the integration of machine learning techniques to augment the predictive power and depth of our analysis. Combining traditional modeling with machine learning algorithms could potentially uncover nuanced patterns and interactions, thereby enriching our understanding of inflation expectations (Bache et al., 2011; Bekiros et al., 2013).

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